
MATH 11010: Graphs of Quad Functions

Section 2.4

Consider the function $f(x) = a(x - h)^2 + k$. This form is often referred to as “standard form” of the quadratic. The graph of this function is a parabola with the following characteristics:

- The vertex of f is (h, k)
- The parabola opens up if $a > 0$, and the parabola opens down if $a < 0$.
- The parabola has $x = h$ as its axis of symmetry.
- The function f has a maximum value of k if $a < 0$.
- The function f has a minimum value of k if $a > 0$.

Example 1: Express $f(x) = 3x^2 + 6x - 7$ in standard form. Identify its vertex, axis of symmetry, and determine whether there is a maximum or minimum value and find that value.

Example 2: Express $f(x) = -4x^2 + 12x - 3$ in standard form. Identify its vertex, axis of symmetry, and determine whether there is a maximum or minimum value and find that value.

Example 3: Express $f(x) = \frac{x^2}{2} - 5x + 1$ in standard form. Identify its vertex, axis of symmetry, and determine whether there is a maximum or minimum value and find that value.

Vertex Formula: The vertex of $f(x) = ax^2 + bx + c$ is

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

Example 4: Consider $f(x) = 5x^2 - 2x + 3$.

- (a) Find the vertex.

- (b) Determine whether there is a maximum or minimum value and find that value.

- (c) Find the range.

- (d) On what intervals is the function increasing? decreasing?

Example 5: A stone is thrown directly upward from a height of 30 feet with an initial velocity of 60 ft/sec. The height of the stone t seconds after it has been thrown is given by the function

$$h(t) = -16t^2 + 60t + 30.$$

Determine the time at which the stone reaches its maximum height and find the maximum height.

Example 6: Ann decides to enclose a rectangular garden, using the side of her garage as one side of the rectangle. What is the maximum area that Ann can enclose with 32 feet of fence? What should the dimensions of the garden be in order to yield this area?

- **Cost Function:** denoted $C(x)$, is the total cost incurred in producing x units of a certain commodity.
- **Revenue:** denoted $R(x)$, is the money received by a company from the sale of x units of a certain commodity.
- **Profit Function:** denoted $P(x)$, is the found by

$$P(x) = R(x) - C(x).$$

Example 7: If x is the number of units sold, then $R(x) = 20x - 0.1x^2$ and $C(x) = 4x + 2$. Find the maximum profit and the maximum number of units that must be sold in order to yield the maximum profit.