MATH 11010: Quadratic Equations
Section 2.3

• **Quadratic Equations:** A quadratic equation is an equation of the form

\[ ax^2 + bx + c = 0, \quad a \neq 0 \]

where \(a\), \(b\), and \(c\) are real numbers.

• **Quadratic Functions:** A quadratic function \(f\) is a function that can be written in the form

\[ f(x) = ax^2 + bx + c, \quad a \neq 0 \]

where \(a\), \(b\), and \(c\) are real numbers.

• **Zeros:** The zeros of a quadratic function \(f(x) = ax^2 + bx + c\) are the solutions of the corresponding quadratic equation \(ax^2 + bx + c = 0\).

• **Zero Product Property:** If \(ab = 0\), then \(a = 0\) or \(b = 0\).

• **Square Root Principle:** If \(x^2 = c\), then \(x = \sqrt{c}\) and \(x = -\sqrt{c}\).

**Example 1:** Solve the following.

(a) \(3(x - 4)^2 - 15 = 0\)  
(b) \(4(x + 2)^2 + 24 = 0\)
<table>
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<th>STEPS FOR COMPLETING THE SQUARE</th>
<th>EXAMPLE: $x^2 + 4x + 2 = 0$</th>
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<tr>
<td>1) Isolate the constant on one side of the equation.</td>
<td>$x^2 + 4x = -2$</td>
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<tr>
<td>2) Make sure the coefficient of $x^2$ is a positive one. If not, divide by this coefficient.</td>
<td>$(\frac{1}{2} \cdot 4)^2 = 2^2 = 4$</td>
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<tr>
<td>3) Determine $(\frac{1}{2} \cdot \text{coeff of } x)^2$</td>
<td>$x^2 + 4x + 4 = -2 + 4$</td>
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<tr>
<td>4) Add the result of step (3) to both sides.</td>
<td>$(x + 2)^2 = 2$</td>
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<tr>
<td>5) Factor as a perfect square and solve by using the Square Root Property.</td>
<td>$\sqrt{(x + 2)^2} = \sqrt{2}$</td>
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<td></td>
<td>$x + 2 = \pm \sqrt{2}$</td>
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<td></td>
<td>$x = -2 \pm \sqrt{2}$</td>
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</table>

**Example 2:** Solve the following by completing the square.

(a) $x^2 + 6x - 5 = 0$  
(b) $2x^2 - 16x + 26 = 0$
The solutions of \( ax^2 + bx + c = 0 \) where \( a \neq 0 \) are given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

- The quadratic formula is a result of solving \( ax^2 + bx + c = 0 \) by completing the square.
- The quadratic formula can be used to solve any quadratic equation.
- The expression \( b^2 - 4ac \) is called the discriminant.

  - If \( b^2 - 4ac < 0 \) then there are two different complex number solutions to the quadratic equation.
  - If \( b^2 - 4ac = 0 \), then the quadratic equation has only one real zero.
  - If \( b^2 - 4ac > 0 \), then the quadratic equation has two different real solutions.

**Example 3:** Solve the following using the quadratic formula.

(a) \( 3x^2 + 4 = 5x \)
(b) \( 4x^2 + 4x - 1 = 0 \)
Example 4: Solve the following.

(a) \( x^4 - 8x^2 = 9 \)

(b) \( x^{1/2} - 4x^{1/4} + 3 = 0 \)

(c) \( (3x + 2)^2 + 7(3x + 2) - 8 = 0 \)

Homework: pp 213–214; #1–91 eoo