MATH 11010: Quadratic Equations Section 2.3

- **Quadratic Equations:** A **quadratic equation** is an equation of the form

$$ax^2 + bx + c = 0, \quad a \neq 0$$

where $a,\ b$, and c are real numbers.

- **Quadratic Functions:** A **quadratic function** f is a function that can be written in the form

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

where $a,\ b$, and c are real numbers.

- **Zeros:** The **zeros** of a quadratic function $f(x) = ax^2 + bx + c$ are the solutions of the corresponding quadratic equation $ax^2 + bx + c = 0$.

- **Zero Product Property:** If $ab = 0$, then $a = 0$ or $b = 0$.

- **Square Root Principle:** If $x^2 = c$, then $x = \sqrt{c}$ and $x = -\sqrt{c}$.

Example 1: Solve the following.

(a) $3(x-4)^2 - 15 = 0$

(b) $4(x+2)^2 + 24 = 0$

STEPS FOR COMPLETING THE SQUARE	EXAMPLE: $x^2+4x+2=0$
1) Isolate the constant on one side of the equation.	$x^2+4x \ = -2$
2) Make sure the coefficient of x^2 is a positive one. If not, divide by this coefficient.	
3) Determine $\left(\frac{1}{2}\cdot \text{ coeff of } x\right)^2$	$\left(\frac{1}{2}\cdot 4\right)^2=2^2=4$
4) Add the result of step (3) to both sides.	$x^2+4x+4=-2+4$
5) Factor as a perfect square and solve by using the Square Root Property.	$(x+2)^2=2$ $\sqrt{(x+2)^2}=\sqrt{2}$ $x+2=\pm\sqrt{2}$ $x=-2\pm\sqrt{2}$

Example 2: Solve the following by completing the square.

(a) $x^2+6x-5=0$

(b) $2x^2-16x+26=0$

Quadratic Formula: The solutions of $ax^2+bx+c=0$ where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

- The quadratic formula is a result of solving $ax^2+bx+c=0$ by completing the square.
- The quadratic formula can be used to solve *any* quadratic equation.
- The expression b^2-4ac is called the **discriminant**.
  - If $b^2-4ac < 0$ then there are two different complex number solutions to the quadratic equation.
  - If $b^2-4ac = 0$, then the quadratic equation has only one real zero.
  - If $b^2-4ac > 0$, then the quadratic equation has two different real solutions.

Example 3: Solve the following using the quadratic formula.

(a) $3x^2+4=5x$ (b) $4x^2+4x-1=0$

Example 4: Solve the following.

(a) $x^4 - 8x^2 = 9$

(b) $x^{1/2} - 4x^{1/4} + 3 = 0$

(c) $(3x + 2)^2 + 7(3x + 2) - 8 = 0$

Homework: pp 213–214; #1–91 eoo