MATH 11010: Rational Functions Section 3.5

- **Rational Function:** A **rational function** is formed by the quotient of two polynomials, $f(x) = \frac{p(x)}{q(x)}$, where $q(x)$ is not the zero polynomial. The domain of f consists of all values of x for which $q(x) \neq 0$.

Example 1: Find the domain of $f(x) = \dfrac{3x+2}{4x^2 - 4x - 15}$.

- **NOTATION:**
  * As x approaches a from the left is denoted $x \to a^-$
  * As x approaches a from the right is denoted $x \to a^+$

- **Vertical Asymptote:** The line $x = a$ is a **vertical asymptote** for the graph of f if any of the following statements are true:

$$f(x) \to \infty \text{ as } x \to a^- \quad \text{or} \quad f(x) \to -\infty \text{ as } x \to a^-$$

$$f(x) \to \infty \text{ as } x \to a^+ \quad \text{or} \quad f(x) \to -\infty \text{ as } x \to a^+$$

Finding Vertical Asymptotes:
For a rational function $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials with no common factors other than constants, if a is a zero of the denominator, then the line $x = a$ is a vertical asymptote for the graph of f.

Example 2: Determine the vertical asymptote(s) for the graph of each of the following functions:

(a) $f(x) = \dfrac{x-4}{5x+3}$

(b) $g(x) = \dfrac{x^2+2x-3}{2x^2-3x-35}$

- **Horizontal Asymptote:** The line $y = b$ is a **horizontal asymptote** for the graph of f if either or both of the following statements are true:

$$f(x) \to b \text{ as } x \to \infty \qquad \text{or} \qquad f(x) \to b \text{ as } x \to -\infty$$

Finding Horizontal Asymptotes: Consider $f(x) = \dfrac{p(x)}{q(x)}$ where p and q are polynomials.

- **Degree of numerator = Degree on denominator:** When the numerator and denominator of a rational function have the same degree, the line $y = \dfrac{a}{b}$ is the horizontal asymptote, where a is the leading coefficient of the numerator and b is the leading coefficient of the denominator.
- **Degree of numerator < Degree of denominator:** When the degree of the numerator is less than the degree of the denominator, the line $y = 0$ (x-axis) is the horizontal asymptote.
- **Degree of numerator > Degree of denominator:** When the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

Example 3: Determine the horizontal asymptote of the graph of the following functions.

(a) $f(x) = \dfrac{5x^2 - 4x + 7}{3x^3 - 2x + 1}$

(b) $f(x) = \dfrac{4x^2 - 3x + 1}{3x^2 - 19x + 2}$

(c) $f(x) = \dfrac{2x^2 - 3x - 1}{x - 2}$

- **Oblique Asymptote:** An oblique asymptote occurs when the degree of the numerator is one more than the degree of the denominator. To find the oblique asymptote, we divide the numerator by the denominator.

Example 4: Determine the oblique asymptote for the graph of the following functions.

(a) $f(x) = \dfrac{2x^2 - 3x - 1}{x - 2}$

(b) $$f(x) = \frac{18x^3 + 6x^2 + 12x + 7}{6x^2 + 5}$$

Example 5: Find a rational function that satisfies the given condition:

Vertical asymptotes at $x = -3,\ x = 5$ and Horizontal asymptote at $y = 2$.

Homework: pp 316–318; 7–25 odd, 27–67 eoo (just find domain and asymptotes), 69, 71