MATH 11010: Symmetry & Transformations Section 1.7

Algebraic Tests for Symmetry:

- **Symmetric with respect to the x-axis:** If replacing y with $-y$ produces an equivalent equation, then the graph is **symmetric with respect to the x-axis**.
- **Symmetric with respect to the y-axis:** If replacing x with $-x$ produces an equivalent equation, then the graph is **symmetric with respect to the y-axis**.
- **Symmetric with respect to the origin:** If replacing x with $-x$ AND y with $-y$ produces an equivalent equation, then the graph is **symmetric with respect to the origin**.

Even and Odd Functions:

- **Even Function:** If the graph of a function f is symmetric with respect to the y-axis, we say that it is an **even function**. That is, for each x in the domain of f, $f(-x) = f(x)$.
- **Odd Function:** If the graph of a function f is symmetric with respect to the origin, we say that it is an **odd function**. That is, for each x in the domain of f, $f(-x) = -f(x)$.

Example 1: Determine if the following functions are even, odd, or neither.

(a) $f(x) = 7x^3 - 5x$

(b) $g(x) = 8x^4 - 2x^2 + 5$

(c) $h(x) = 9x^5 - 3x + 1$

Transformations of functions:

- **Vertical Shift**: Suppose that $c > 0$.
  - The equation $y = f(x) + c$ shifts the graph of $y = f(x)$ up c units. (Adding a constant on the outside of a functions shifts the graph up.)
  - The equation $y = f(x) - c$ shifts the graph of $y = f(x)$ down c units. (Subtracting a constant on the outside of a function shifts the graph down.)

- **Horizontal Shift**: Suppose that $c > 0$.
  - The equation $y = f(x+c)$ shifts the graph of $y = f(x)$ to the left c units. (Adding a constant inside the function shifts the graph left.)
  - The equation $y = f(x - c)$ shifts the graph of $y = f(x)$ to the right c units. (Subtracting a constant inside the function shifts the graph right.)

- **Reflections**:
  - The equation $y = -f(x)$ reflects the graph of $y = f(x)$ with respect to the $x-$axis. (Multiplying by a negative on the outside of a function flips the graph with respect to the $x-$axis.)
  - The equation $y = f(-x)$ reflects the graph of $y = f(x)$ with respect to the the $y-$axis. (Multiplying by a negative inside the function flips the graph with respect to the $y-$axis.)

- **Vertical Stretching and Shrinking**:
  - When $c > 1$, the equation $y = cf(x)$ stretches the graph of $y = f(x)$ vertically by a factor of c. (Multiplying by a number, larger than one, on the outside of a function causes the graph to be stretched or narrowed by a factor of c.)
  - When $0 < c < 1$, the equation $y = cf(x)$ shrinks the graph of $y = f(x)$ vertically by a factor of c. (Multiplying by a number, between zero and one, on the outside of a function causes the graph to shrink or widen by a factor of c.)

- **Horizontal Stretching and Shrinking**:
  - When $c > 1$, the equation $y = f(cx)$ shrinks the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$. (Multiplying by a number, larger than one, on the inside of the function causes the graph to shrink horizontally by a factor of $\frac{1}{c}$.)
  - When $0 < c < 1$, the equation $y = f(cx)$ stretches the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$. (Multiplying by a number, between zero and one, on the inside of the function causes the graph to be stretched by a factor of $\frac{1}{c}$.)

Example 2: Determine how the graph of

$$y = -\frac{9}{2}f(x+2) - 9$$

can be obtained from the graph of f. Be specific.

Example 3: A function f is given, and the indicated transformations are applied to its graph in the given order. Write the equation for the final transformed graph.

(a) $f(x) = \sqrt{x}$; reflected about the $y-$axis, shrunk horizontally by a factor of $\frac{1}{9}$, vertical shift down 8 units.

(b) $f(x) = x^2$; reflected about the $x-$axis, vertically stretched by a factor of 8, horizontal shift left 5 units

Example 4: The point $(-12, 4)$ is on the graph of $y = f(x)$. Find a point on the graph of $y = g(x)$.

(a) $g(x) = f(x-5)$ (b) $g(x) = f(4x)$ (c) $g(x) = -2f(x)$

Homework: pp 163–165; #7–105 eoo