MATH 11010: Symmetry & Transformations
Section 1.7

Algebraic Tests for Symmetry:

- **Symmetric with respect to the x-axis:** If replacing $y$ with $-y$ produces an equivalent equation, then the graph is **symmetric with respect to the x-axis.**

- **Symmetric with respect to the y-axis:** If replacing $x$ with $-x$ produces an equivalent equation, then the graph is **symmetric with respect to the y-axis.**

- **Symmetric with respect to the origin:** If replacing $x$ with $-x$ AND $y$ with $-y$ produces an equivalent equation, then the graph is **symmetric with respect to the origin.**

Even and Odd Functions:

- **Even Function:** If the graph of a function $f$ is symmetric with respect to the $y$-axis, we say that it is an **even function.** That is, for each $x$ in the domain of $f$, $f(-x) = f(x)$.

- **Odd Function:** If the graph of a function $f$ is symmetric with respect to the origin, we say that it is an **odd function.** That is, for each $x$ in the domain of $f$, $f(-x) = -f(x)$.

**Example 1:** Determine if the following functions are even, odd, or neither.

(a) $f(x) = 7x^3 - 5x$

(b) $g(x) = 8x^4 - 2x^2 + 5$

(c) $h(x) = 9x^5 - 3x + 1$
Transformations of functions:

- **Vertical Shift**: Suppose that $c > 0$.
  - The equation $y = f(x) + c$ shifts the graph of $y = f(x)$ up $c$ units. (Adding a constant on the outside of a function shifts the graph up.)
  - The equation $y = f(x) - c$ shifts the graph of $y = f(x)$ down $c$ units. (Subtracting a constant on the outside of a function shifts the graph down.)

- **Horizontal Shift**: Suppose that $c > 0$.
  - The equation $y = f(x + c)$ shifts the graph of $y = f(x)$ to the left $c$ units. (Adding a constant inside the function shifts the graph left.)
  - The equation $y = f(x - c)$ shifts the graph of $y = f(x)$ to the right $c$ units. (Subtracting a constant inside the function shifts the graph right.)

- **Reflections**:
  - The equation $y = -f(x)$ reflects the graph of $y = f(x)$ with respect to the $x$–axis. (Multiplying by a negative on the outside of a function flips the graph with respect to the $x$–axis.)
  - The equation $y = f(-x)$ reflects the graph of $y = f(x)$ with respect to the $y$–axis. (Multiplying by a negative inside the function flips the graph with respect to the $y$–axis.)

- **Vertical Stretching and Shrinking**:
  - When $c > 1$, the equation $y = cf(x)$ stretches the graph of $y = f(x)$ vertically by a factor of $c$. (Multiplying by a number, larger than one, on the outside of a function causes the graph to be stretched or narrowed by a factor of $c$.)
  - When $0 < c < 1$, the equation $y = cf(x)$ shrinks the graph of $y = f(x)$ vertically by a factor of $c$. (Multiplying by a number, between zero and one, on the outside of a function causes the graph to shrink or widen by a factor of $c$.)

- **Horizontal Stretching and Shrinking**:
  - When $c > 1$, the equation $y = f(cx)$ shrinks the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$. (Multiplying by a number, larger than one, on the inside of the function causes the graph to shrink horizontally by a factor of $\frac{1}{c}$.)
  - When $0 < c < 1$, the equation $y = f(cx)$ stretches the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$. (Multiplying by a number, between zero and one, on the inside of the function causes the graph to be stretched by a factor of $\frac{1}{c}$.)
Example 2: Determine how the graph of
\[ y = -\frac{9}{2} f(x + 2) - 9 \]
can be obtained from the graph of \( f \). Be specific.

Example 3: A function \( f \) is given, and the indicated transformations are applied to its graph in the given order. Write the equation for the final transformed graph.

(a) \( f(x) = \sqrt{x} \); reflected about the \( y \)-axis, shrunk horizontally by a factor of \( \frac{1}{9} \), vertical shift down 8 units.

(b) \( f(x) = x^2 \); reflected about the \( x \)-axis, vertically stretched by a factor of 8, horizontal shift left 5 units

Example 4: The point \((-12, 4)\) is on the graph of \( y = f(x) \). Find a point on the graph of \( y = g(x) \).

(a) \( g(x) = f(x - 5) \)  
(b) \( g(x) = f(4x) \)  
(c) \( g(x) = -2f(x) \)

Homework: pp 163–165; #7–105 eoo