NAME: Key

MATH 11012 EXAM #3 (180 pts) **FALL 2012**

SHOW ALL WORK FOR FULL CREDIT — PLEASE CIRCLE YOUR FINAL ANSWER
SIMPLIFY ALL ANSWERS – EXACT ANSWERS ONLY

1. (4 pts) Find the relative extrema of

$$f(x) = 2x^5 - 20x^4 + 27$$

Give only the x–coordinate(s) of the extrema. Indicate your answer as a relative maximum or minimum. Pay close attention to the function's domain and its vertical asymptotes (if any). Be sure to show all work – ANSWERS ALONE ARE UNACCEPTABLE.

$f'(x) = 10x^4 - 80x^3$

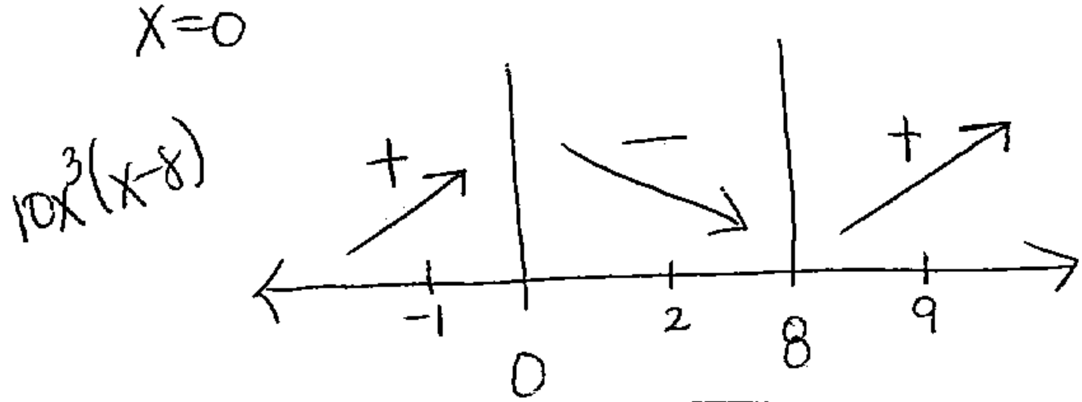
$10x^4 - 80x^3 = 0$

$10x^3(x-8) = 0$

$10x^3 = 0 \qquad x - 8 = 0$

$x^3 = 0 \qquad x = 8$

$x = 0$



Rel max at $x=0$
Rel. min at $x=8$

Exam Score: Course Grade:

$\frac{\quad}{40} = \frac{\quad}{180}$ $\frac{\quad}{555} =$

2. (4 pts) Find the relative extrema of

$$f(x) = 4x^3e^{2x}$$

Give only the x−coordinate(s) of the extrema. Indicate your answer as a relative maximum or minimum. Pay close attention to the function's domain and its vertical asymptotes (if any). Be sure to show all work – ANSWERS ALONE ARE UNACCEPTABLE.

$$f'(x) = 4x^3 \cdot 2e^{2x} + e^{2x} \cdot 12x^2$$

$$f'(x) = 8x^3e^{2x} + 12x^2e^{2x}$$

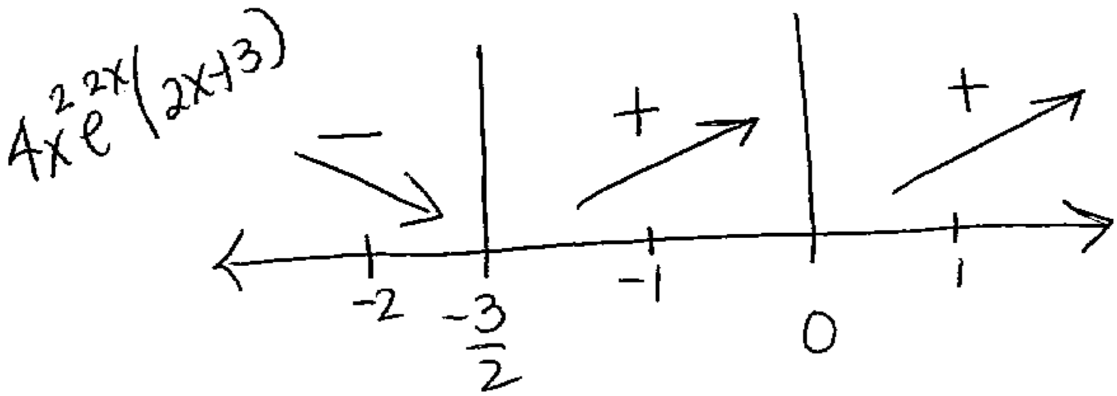
$$8x^3e^{2x} + 12x^2e^{2x} = 0$$

$$4x^2e^{2x}(2x+3) = 0$$

$4x^2 = 0$ $\quad$ $e^{2x} = 0$ $\quad$ $2x+3=0$

$x^2 = 0$ $\quad$ $\ln e^{2x} \neq \ln 0$ $\quad$ $2x = -3$

$x = 0$ $\quad\quad$ $x = \frac{-3}{2}$



$$\boxed{\text{Rel min at } x = \frac{-3}{2}}$$

3. (4 pts) Find the relative extrema of

$$f(x) = \frac{3x^2}{x-5}$$

V.A. at $X=5$

Give only the x−coordinate(s) of the extrema. Indicate your answer as a relative maximum or minimum. Pay close attention to the function's domain and its vertical asymptotes (if any). Be sure to show all work – ANSWERS ALONE ARE UNACCEPTABLE.

$$f'(x) = \frac{(x-5)(6x) - 3x^2(1)}{(x-5)^2} = \frac{6x^2 - 30x - 3x^2}{(x-5)^2}$$

$$f'(x) = \frac{3x^2 - 30x}{(x-5)^2} = \frac{3x(x-10)}{(x-5)^2}$$

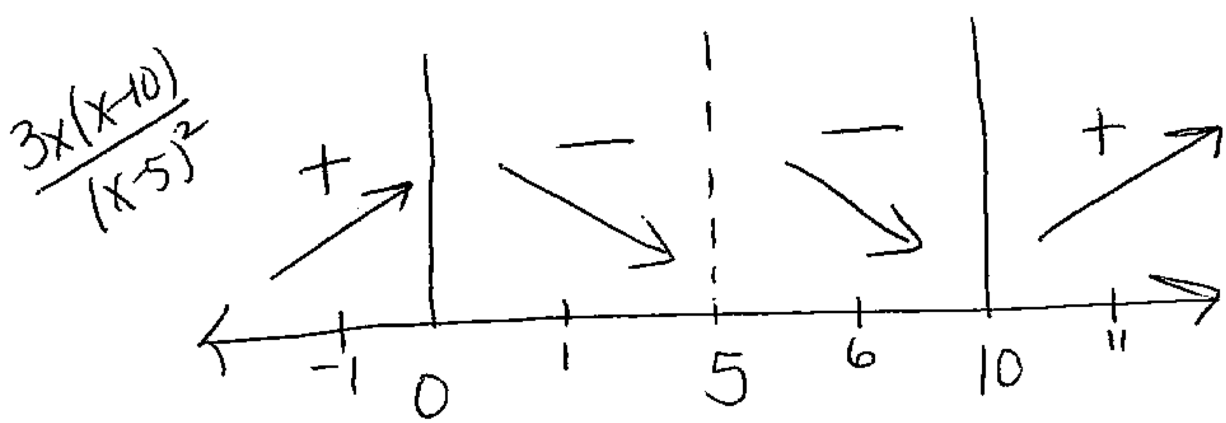
$3x = 0$

$x = 0$

$x - 10 = 0$

$x = 10$

Vertical asym at $x=5$



$\boxed{\text{Rel max at } x=0 \quad \text{Rel min at } x=10}$

4. (4 pts) A manufacturer of fishing poles finds that the total cost in dollars of manufacturing x poles per day is given by

$$C(x) = 300 + 2x + 0.0001x^2.$$

Each pole can be sold at a price of p dollars, where p is related to x by the demand equation

$$p = 12 - 0.0004x.$$

If all fishing poles that are manufactured can be sold, find the daily level of production that will yield a maximum profit for the manufacturer. You must justify your answer.

$$R(x) = xp = 12x - 0.0004x^2$$

$$P(x) = R(x) - C(x) = 12x - 0.0004x^2 - 300 - 2x - 0.0001x^2$$

$$P(x) = -300 + 10x - 0.0005x^2$$

$$P'(x) = 10 - 0.001x$$

$$10 - 0.001x = 0$$

$$\frac{10}{0.001} = \frac{0.001x}{0.001}$$

$$10{,}000 = x$$

$$P''(x) = -0.001$$

$P''(10{,}000) = -0.001 < 0$ max

$\boxed{10{,}000 \text{ poles}}$

5. (1.5 pts) The current i (in amperes A) in a certain circuit at a given time t (in seconds) is given by the equation

$$i = 0.14t^2 \ln(5t + 4) \qquad 0 \leq t \leq 8.$$

Find the instantaneous rate of change of i with respect to time when $t = 2$ seconds. Round your answer to two decimal places. Your answer must have the proper units to receive full credit.

$$i'(t) = 0.14t^2\left(\frac{5}{5t+4}\right) + 0.28t\ \ln(5t+4)$$

$$i'(2) = 0.14(2)^2\left(\frac{5}{10+4}\right) + 0.28(2)\ln(5 \cdot 2+4)$$

$$i'(2) = 0.56\left(\frac{5}{14}\right) + 0.56\ln(14)$$

$$= 0.2 + 0.56 \ln 14$$

$$= \boxed{1.68 \text{ amps/sec}}$$

6. (2.5 pts) Given $f(x) = \frac{3}{x^2+7}$, find and simplify $f''(x)$.

$$f'(x) = \frac{(x^2+7)(0) - 3(2x)}{(x^2+7)^2} = \frac{-6x}{(x^2+7)^2}$$

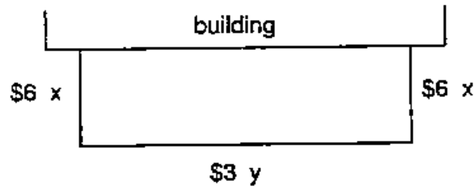
$$f''(x) = \frac{(x^2+7)^2(-6) - (-6x)\cdot 2(x^2+7)(2x)}{(x^2+7)^4}$$

$$= \frac{-6(x^2+7)^2 + 24x^2(x^2+7)}{(x^2+7)^4}$$

$$= \frac{-6(x^2+7)\left[x^2+7-4x^2\right]}{(x^2+7)^4}$$

$$f''(x) = \boxed{\frac{-6(7-3x^2)}{(x^2+7)^3}}$$

7. (5 pts) The management of a department store has decided to enclose an 900 ft^2 area outside the building for displaying potted plants and flowers. One side will be formed by the external wall of the store, the two parallel sides will be constructed of galvanized steel fencing costing \$6 per foot, and the fourth side will be made of pine board fencing costing \$3 per foot. Determine the dimensions of the enclosure that can be constructed at minimum cost.



$$A = xy$$

$$900 = xy \Rightarrow y = \frac{900}{x}$$

$$\text{Cost} = 6x + 3y + 6x = 12x + 3y$$

$$C(x) = 12x + 3\left(\frac{900}{x}\right)$$

$$C(x) = 12x + \frac{2700}{x}$$

$$C'(x) = 12 - \frac{2700}{x^2}$$

$$12 - \frac{2700}{x^2} = 0$$

$$12 = \frac{2700}{x^2}$$

$$\frac{12x^2}{12} = \frac{2700}{12}$$

$$\sqrt{x^2} = \sqrt{225}$$

$$x = 15$$

$$C''(x) = \frac{5400}{x^3}$$

$$C''(15) = \frac{5400}{15^3} > 0$$

min.

$$y = \frac{900}{15} = 60$$

$$\boxed{15' \times 60'}$$

8. The university is trying to determine what price to charge for football tickets. If the price per ticket is \$7.50, then 70,000 people will attend the game. For every decrease of \$1, the attendance increases by 10,000 people.

(a) (4 pts) What ticket price should be charges in order to maximize revenue?

Let $x = \#$ \$1 price decreases

price $= p(x) = 7.50 - 1x$

quantity $= q(x) = 70{,}000 + 10{,}000x$

$R(x) = (7.50 - x)(70{,}000 + 10{,}000x)$

$R(x) = 525000 + 75000x - 70000x - 10000x^2$

$R(x) = 525000 + 5000x - 10000x^2$

$R'(x) = 5000 - 20000x$

$5000 - 20000x = 0$

$\frac{5000}{20000} = \frac{20000x}{20000}$

$0.25 = x$

$R''(x) = -20{,}000$

$R''(0.25) = -20{,}000 < 0$ max

price $= 7.50 - 0.25$
$= \boxed{\$7.25}$

(b) (0.5 pt) How many people will attend at the price found in part (a)?

quantity $= 70{,}000 + 10{,}000(0.25)$
$= 70{,}000 + 2500$
$= \boxed{72{,}500}$

9. (4 pts) The tensile strength S (in Newtons N) of a certain material as a function of the temperature T (in degrees Celsius) is

$$S = 6\sqrt{T^2 - 4T + 50} \qquad 0 \leq T \leq 30.$$

At what temperature is the tensile strength a minimum? Round your answer to two decimal places. You must verify your answer.

$$S = 6(T^2 - 4T + 50)^{1/2}$$

$$S'(T) = 3(T^2 - 4T + 50)^{-1/2}(2T - 4)$$

$$S'(T) = \frac{3(2T-4)}{\sqrt{T^2 - 4T + 50}}$$

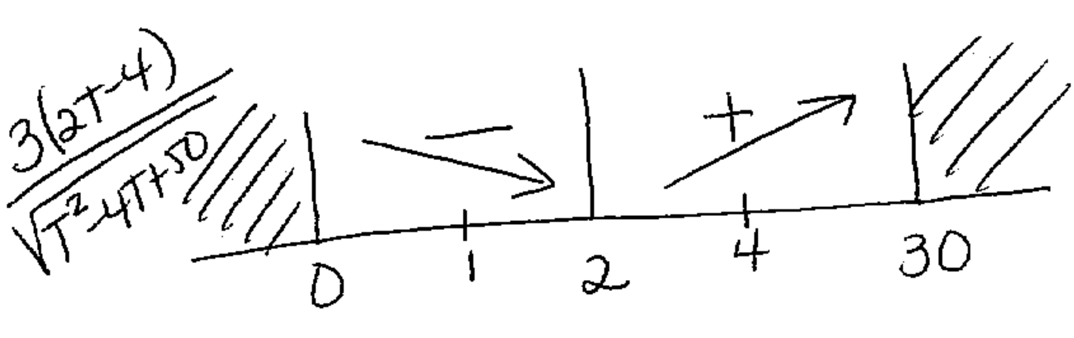
$$\frac{3(2T-4)}{\sqrt{T^2-4T+50}} = 0$$

$$3(2T-4) = 0$$

$$3 \neq 0 \qquad 2T - 4 = 0$$

$$2T = 4$$

$$T = 2$$



therefore, $T=2$ is where rel min occurs.

$$\boxed{T = 2^\circ C}$$

10. Consider $f(x) = 3x^3 - 18x^2 - 45x + 20$.

(a) (0.5 pt) Find the y-intercept.

$\boxed{(0, 20)}$

(b) (1.5 pts) Determine the intervals where f is increasing and the intervals where f is decreasing (label answers)

$f'(x) = 9x^2 - 36x - 45 = 9(x^2 - 4x - 5)$

$9(x^2 - 4x - 5) = 0$

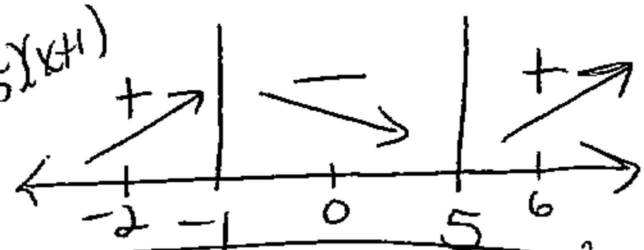
$9(x-5)(x+1) = 0$

$9 \neq 0 \quad x - 5 = 0 \quad x + 1 = 0$

$x = 5 \quad x = -1$

$9(x-5)(x+1)$

$+$



$\boxed{\begin{array}{l} \text{inc}: (-\infty, -1) \cup (5, \infty) \\ \text{dec}: (-1, 5) \end{array}}$

(c) (1 pt) Determine the relative maximum(s) and relative minimum(s) (label answers)

$\boxed{\begin{array}{l} \text{Rel max} = (-1, 44) \\ \text{Rel min} = (5, -280) \end{array}}$

$f(-1) = -3 - 18 + 45 + 20$

$f(5) = 3(125) - 18(25) - 45(5) + 20$

(d) (1 pt) Determine the intervals where f is concave up and the intervals where f is concave down (label answers)

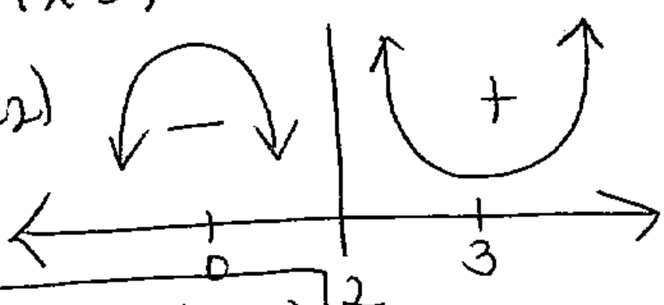
$f''(x) = 18x - 36 = 18(x-2)$

$18(x-2) = 0$

$18 \neq 0 \quad x - 2 = 0$

$x = 2$

$18(x-2)$



$\boxed{\begin{array}{l} \text{Concave up}: (2, \infty) \\ \text{Concave down}: (-\infty, 2) \end{array}}$

(e) (0.5 pt) Determine the point(s) of inflection

$\boxed{(2, -118)}$

$f(2) = 3(8) - 18(4) - 45(2) + 20$

$= -118$

(f) (2 pts) Sketch a complete graph of f. Be sure you include all points of interest. Your graph does not have to be to scale.