1. Find the derivative of the following functions.

(a) \( f(x) = \frac{x^4}{8} - 7x^{-5} + 3x^{-2/3} \)

(b) \( g(x) = \frac{3}{x^2} - 15\sqrt[3]{x^3} + 5x^{-4} \)

(c) \( f(x) = (3x - 7)(5x^4 + 3x^2 - 7)^5 \)

(d) \( g(x) = \frac{x^6 + 3x^2 - 1}{5x^4 + 2x} \)

(e) \( f(x) = \sqrt{\frac{5x - 7}{6x^2 + x - 2}} \)

(f) \( g(x) = (3x^2 + 7x - 2)(6x^3 + 2x^2 + 3) \)

2. Find the following limits.

(a) \( \lim_{x \to -2} (3x^2 - 5x + 1) \)

(b) \( \lim_{x \to 2} \frac{x^2 + 7}{3x + 1} \)

(c) \( \lim_{x \to 1} \frac{x^2 - x}{x^2 - 1} \)

(d) \( \lim_{h \to 0} \frac{4x^2h + xh^2 - h^3}{h} \)

3. Answer the following given

\[ f(x) = \begin{cases} 
5 - x, & \text{if } x < 4 \\
2x - 5, & \text{if } x \geq 4 
\end{cases} \]

(a) \( \lim_{x \to 4^+} f(x) \)

(b) \( \lim_{x \to 4^-} f(x) \)

(c) \( \lim_{x \to 4} f(x) \)

4. Determine whether each of the following functions is continuous or discontinuous at \( x = -1 \). If it is discontinuous, indicate the first of the three conditions in the definition of continuity that is violated.

\[ y = \begin{cases} 
1/x, & \text{if } x < 0 \\
0, & \text{if } x = 0 \\
-x, & \text{if } x > 0 
\end{cases} \]
5. Given the function $f$ graphed below, answer the following

![Graph of function f]

(a) \[ \lim_{x \to -2^+} f(x) \]
(b) \[ \lim_{x \to -2^-} f(x) \]
(c) \[ \lim_{x \to -2} f(x) \]
(d) \[ \lim_{x \to 1} f(x) \]
(e) List all values of $x$ where $f$ is not differentiable.

6. Find the equation of the tangent line to $f(x) = 2x^2 - x + 4$ at $x = 3$.

7. If $f(x) = (x^2 - 1)(x^2 + x)$ find $f''(2)$

8. For the function $f(x) = 3x^2 + 5x - 8$, find $f'(x)$ using the DEFINITION of the derivative.

9. Determine whether $f(x) = \frac{(x - 1)(x + 2)}{x - 3}$ is continuous or discontinuous. If discontinuous, state where it is discontinuous.

10. A rocket rises to a height of $s(t) = 10t^2 + 9t^{4/3}$ feet in $t$ seconds. Find the rockets velocity and acceleration at time $t = 8$ seconds.

11. A steel mill finds that its cost function is $C(x) = 8000\sqrt{x} - 6000\sqrt[3]{x}$ dollars, where $x$ is the daily production of steel in tons.

(a) Find the marginal cost function.

(b) Find the marginal cost when 64 tons of steel are produced and interpret your answer.
ANSWERS

1. (a) \( f'(x) = \frac{1}{2}x^3 + 35x^{-6} - 2x^{-5/3} \)

(b) \( g'(x) = -6x^{-3} - 9x^{-2/5} - 20x^{-5} \)

(c) \( f'(x) = (3x - 7) \cdot 5(5x^4 + 3x^2 - 7)^4(20x^3 + 6x) + (5x^4 + 3x^2 - 7)^5(3) \)

(d) \( g'(x) = \frac{(5x^4 + 2x)(6x^5 + 6x) - (x^6 + 3x^2 - 1)(20x^3 + 2)}{(5x^4 + 2x)^2} \)

(e) \( f'(x) = \frac{1}{2} \left( \frac{5x - 7}{6x^2 + x - 2} \right)^{-1/2} \left( \frac{(6x^2 + x - 2)(5) - (5x - 7)(12x + 1)}{(6x^2 + x - 2)^2} \right) \)

(f) \( g'(x) = (3x^2 + 7x - 2)(18x^2 + 4x) + (6x^3 + 2x^2 + 3)(6x + 7) \)

2. (a) 23

(b) \( \frac{11}{7} \)

(c) \( \frac{1}{2} \)

(d) \( 4x^2 \)

3. (a) 3

(b) 1

(c) \( \text{dne} \)

4. (a) discontinuous; \( \lim_{x \to -1} f(x) \) \( \text{dne} \)

(b) discontinuous; \( f(-1) \neq \lim_{x \to -1} f(x) \)

(c) discontinuous; \( f(-1) \) undefined

5. (a) 4

(b) 2

(c) \( \text{dne} \)
(d) 0
(e) −2, 1

6. $y = 11x - 14$

7. $f''(x) = 12x^2 + 6x - 2$, $f''(2) = 58$

8. $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = 6x + 5$

9. discontinuous at $x = 3$

10. $v(t) = 20t + 12t^{1/3}$; $v(8) = 184$ ft/sec
    $a(t) = 20 + 4t^{-2/3}$; $a(8) = 21$ ft/sec$^2$

11. (a) $C''(x) = 4000x^{-1/2} - 2000x^{-2/3}$
    (b) $C'(64) = 375$.

    Interpretation: After producing 64 tons, the cost to produce the 65th ton is $375.$