
Sections 5.1 & 5.2: Antiderivatives and Indefinite Integrals

Definition. A function F is called an **antiderivative** of f on an interval if $F'(x) = f(x)$ for all x in that interval.

Result. If F is an antiderivative of f on an interval, then the most general antiderivative of f on that interval is

$$F(x) + C,$$

where C is an arbitrary constant.

Definition. The set of all antiderivatives of the function f on an interval is called the **indefinite integral of f** and is denoted $\int f(x) dx$ (read “the integral of $f(x)$, dx ”). Thus,

$$\int f(x) dx = F(x) + C,$$

where F is an antiderivative of f and C is an arbitrary constant called the **constant of integration**. The symbol “ \int ” is called the **integral sign** and $f(x)$ is called the **integrand**. The symbol “ dx ” has no official meaning by itself — its purpose is to show which variable the integral is with respect to. Note that *the integral sign must always be written with dx* .

Definition. The process of finding an antiderivative is called **integration**.

Example 1. Find $\int x^7 dx$.

SOLUTION. An antiderivative of x^7 is $\frac{1}{8}x^8$. Hence, $\int x^7 dx = \boxed{\frac{1}{8}x^8 + C}$.

Notice that we can check this result by differentiating:

$$F(x) = \frac{1}{8}x^8 + C$$

$$F'(x) = x^7$$

(The derivative of the constant C is just zero.)

When calculating an indefinite integral, it is very easy to check your answer. You simply take the derivative of your final result — it should be the original function (i.e., the integrand).

BASIC INTEGRATION FORMULAS

- $\int 1 \, dx = x + C$
- $\int k \, dx = kx + C,$ where k is any constant
- $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C,$ for $n \neq -1$
- $\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln |x| + C$
- $\int e^x \, dx = e^x + C$
- $\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C,$ where k is any constant
- $\int kf(x) \, dx = k \int f(x) \, dx,$ where k is any constant
- $\int [f(x) \pm g(x) \, dx] = \int f(x) \, dx \pm \int g(x) \, dx$

Example 2. Find $\int (4x^3 + 6x^2 - 7x + 3) \, dx.$

Example 3. Find $\int (3x^{1/2} + 2x^{2/3} + 4x^{-2} + 3x^{-1/2}) dx.$

Example 4. Find $\int (6\sqrt{x} - 5\sqrt[3]{x} + 2\sqrt[3]{x^2} - 3) dx.$

Example 5. Find $\int \left(\frac{4}{x} + \frac{5}{x^2} + \frac{6}{x^3} + \frac{7}{3\sqrt{x}} \right) dx.$

Example 6. Find $\int \left(\frac{3}{x} + e^{2x} + 5e^{-4x} - 7e^{3x} \right) dx.$

Example 7. Find $\int \frac{x^2 - 5x + 2}{x} dx.$

Example 8. Find $\int \frac{9x^3 + 8x^2 + 3x - 4}{3x^3} dx.$

EXERCISES

Find the following integrals. Check your answer by differentiating.

1. $\int (6x^2 - 4x + 3) dx$

11. $\int (7e^{3x} - 2x^{-1} + x + 3) dx$

2. $\int (4x^3 - 7x^2 + 3x - 5) dx$

12. $\int (2e^{-3x} - 4x^{-1} + 5) dx$

3. $\int (2x^4 + 3x^3 - 5x^2 - 2x + 1) dx$

13. $\int (2e^{-x} + 3e^x + x^2 - 6) dx$

4. $\int (2x^4 + 6x^3 + 7x^2 - 4x + 1) dx$

14. $\int \left(4\sqrt{x} + 3\sqrt[3]{x^2} - \frac{4}{x^2} - \frac{3}{x} + 2 \right) dx$

5. $\int (5x^2 - 7x + 6) dx$

15. $\int \left(\frac{5}{x^3} - \frac{2}{x} + \frac{3}{\sqrt{x}} + x + 1 \right) dx$

6. $\int (6x^3 - 6x^2 + 8x - 2) dx$

16. $\int \left(\frac{1}{e^x} + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{\sqrt{x}} + 3 \right) dx$

7. $\int (3x^{-2} + 4x^{-3} + 6) dx$

17. $\int \frac{x^2 + 5x + 1}{x} dx$

8. $\int (5x^{3/2} + 2x^{1/2} - 7x + 3) dx$

18. $\int \frac{3x^3 - 7x^2 + 4x - 1}{x} dx$

9. $\int (7x^{-3} + 3x^{-1/2} - 5x^{-3/2} - 6) dx$

19. $\int \frac{2x^2 - 3x + 5}{x^2} dx$

10. $\int (3x^{-1} + 4e^{2x} + 5x - 6) dx$

20. $\int \frac{4x^3 - 8x^2 + 3x - 4}{2x^2} dx$

ANSWERS

1. $2x^3 - 2x^2 + 3x + C$

11. $\frac{7}{3}e^{3x} - 2 \ln|x| + \frac{1}{2}x^2 + 3x + C$

2. $x^4 - \frac{7}{3}x^3 + \frac{3}{2}x^2 - 5x + C$

12. $-\frac{2}{3}e^{-3x} - 4 \ln|x| + 5x + C$

3. $\frac{2}{5}x^5 + \frac{3}{4}x^4 - \frac{5}{3}x^3 - x^2 + x + C$

13. $-2e^{-x} + 3e^x + \frac{1}{3}x^3 - 6x + C$

4. $\frac{2}{5}x^5 + \frac{3}{2}x^4 + \frac{7}{3}x^3 - 2x^2 + x + C$

14. $\frac{8}{3}x^{3/2} + \frac{9}{5}x^{5/3} + 4x^{-1} - 3 \ln|x| + 2x + C$

5. $\frac{5}{3}x^3 - \frac{7}{2}x^2 + 6x + C$

15. $-\frac{5}{2}x^{-2} - 2 \ln|x| + 6x^{1/2} + \frac{1}{2}x^2 + x + C$

6. $\frac{3}{2}x^4 - 2x^3 + 4x^2 - 2x + C$

16. $-e^{-x} + \ln|x| - x^{-1} + 2x^{1/2} + 3x + C$

7. $-3x^{-1} - 2x^{-2} + 6x + C$

17. $\frac{1}{2}x^2 + 5x + \ln|x| + C$

8. $2x^{5/2} + \frac{4}{3}x^{3/2} - \frac{7}{2}x^2 + 3x + C$

18. $x^3 - \frac{7}{2}x^2 + 4x - \ln|x| + C$

9. $-\frac{7}{2}x^{-2} + 6x^{1/2} + 10x^{-1/2} - 6x + C$

19. $2x - 3 \ln|x| - 5x^{-1} + C$

10. $3 \ln|x| + 2e^{2x} + \frac{5}{2}x^2 - 6x + C$

20. $x^2 - 4x + \frac{3}{2} \ln|x| + 2x^{-1} + C$