
Section 5.6: Integration by Substitution

If the integrand has a quantity inside a set of parentheses or under a radical, you typically perform the integration using a u -substitution. The usual choice for u will be the quantity inside the parentheses. Be sure to account for the derivative du .

Example 1. Find $\int 15x^2(5x^3 + 6)^4 dx$.

SOLUTION. We use a u -substitution with $u = 5x^3 + 6$:

$$\begin{aligned}\int 15x^2(5x^3 + 6)^4 dx &= \int u^4 du \\ \begin{aligned} u &= 5x^3 + 6 \\ du &= 15x^2 dx \end{aligned} &= \frac{1}{5}u^5 + C \\ &= \boxed{\frac{1}{5}(5x^3 + 6)^5 + C}\end{aligned}$$

Example 2. Find $\int (9x - 5)^3 dx$.

SOLUTION. We use a u -substitution with $u = 9x - 5$:

$$\begin{aligned}\int (9x - 5)^3 dx &= \int u^3 \cdot \frac{1}{9} du \\ \begin{aligned} u &= 9x - 5 \\ du &= 9 dx \\ \frac{1}{9} du &= dx \end{aligned} &= \frac{1}{9} \int u^3 du \\ &= \frac{1}{9} \left(\frac{1}{4} u^4 \right) + C \\ &= \frac{1}{36} u^4 + C \\ &= \boxed{\frac{1}{36} (9x - 5)^4 + C}\end{aligned}$$

Example 3. Find $\int x^2 \sqrt{2x^3 + 5} dx$.

Example 4. Find $\int e^{9x-5} dx$.

Example 5. Find $\int (3x^2 + 2)(x^3 + 2x + 1)^5 dx$

Example 6. Find $\int (x + 3)\sqrt{x^2 + 6x + 1} dx$

Example 7. Find $\int \frac{1}{(7x + 4)^3} dx$

Example 8. Find $\int \frac{x + 3}{(x^2 + 6x - 3)^5} dx$

Example 9. Find $\int 9x(x^2 - 6)^5 dx$

Example 10. Find $\int (x + 1)(4x^2 + 8x - 9)^3 dx$

Example 11. Find $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

Example 12. Find $\int \frac{(e^{-x} + 5)^3}{e^x} dx$

Example 13. Find $\int \frac{4x - 3}{2x^2 - 3x + 7} dx$

Example 14. Find $\int \frac{x - 2}{4x^2 - 16x + 3} dx$

Example 15. Find $\int_{-2}^3 (x-1)^3 dx$

Example 16. Find $\int_{-1}^1 x\sqrt{x^2+8} dx$

Example 17. Find $\int_3^4 \frac{1}{2-x} dx$

EXERCISES

Find the following integrals. Check your answer by differentiating.

1. $\int 2x(x^2 + 3)^4 dx$

14. $\int \frac{1}{\sqrt[3]{4x+1}} dx$

2. $\int (3x^2 + 2)(x^3 + 2x + 1)^5 dx$

15. $\int (x + 1)(3x^2 + 6x - 1)^3 dx$

3. $\int (4x - 5)^8 dx$

16. $\int \frac{1}{\sqrt{3x+5}} dx$

4. $\int \sqrt{5x+3} dx$

17. $\int e^{4x-3} dx$

5. $\int x\sqrt{x^2+4} dx$

18. $\int xe^{x^2} dx$

6. $\int (x+2)\sqrt{x^2+4x+1} dx$

19. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

7. $\int \frac{1}{(5x-2)^3} dx$

20. $\int \frac{(e^{-x}+2)^6}{e^x} dx$

8. $\int \frac{x}{(x^2+6)^3} dx$

21. $\int_{-1}^0 (x+1)^3 dx$

9. $\int \frac{x+3}{(x^2+6x-3)^5} dx$

22. $\int_0^2 (x-2)^2 dx$

10. $\int 2x(3x^2-1)^4 dx$

23. $\int_0^4 \sqrt{2x+1} dx$

11. $\int 5x(x^2+3)^5 dx$

24. $\int_{-1}^1 x\sqrt{x^2+3} dx$

12. $\int \sqrt[3]{8x-1} dx$

25. $\int_0^1 x^2(x^3+2)^2 dx$

13. $\int x^2(x^3-6)^4 dx$

26. $\int_0^2 x^2\sqrt{x^3+1} dx$

ANSWERS

1. $\frac{1}{5}(x^2 + 3)^5 + C$

2. $\frac{1}{6}(x^3 + 2x + 1)^6 + C$

3. $\frac{1}{36}(4x - 5)^9 + C$

4. $\frac{2}{15}(5x + 3)^{3/2} + C$

5. $\frac{1}{3}(x^2 + 4)^{3/2} + C$

6. $\frac{1}{3}(x^2 + 4x + 1)^{3/2} + C$

7. $-\frac{1}{10}(5x - 2)^{-2} + C$

8. $-\frac{1}{4}(x^2 + 6)^{-2} + C$

9. $-\frac{1}{8}(x^2 + 6x - 3)^{-4} + C$

10. $\frac{1}{15}(3x^2 - 1)^5 + C$

11. $\frac{5}{12}(x^2 + 3)^6 + C$

12. $\frac{3}{32}(8x - 1)^{4/3} + C$

13. $\frac{1}{15}(x^3 - 6)^5 + C$

14. $\frac{3}{8}(4x + 1)^{2/3} + C$

15. $\frac{1}{24}(3x^2 + 6x - 1)^4 + C$

16. $\frac{2}{3}(3x + 5)^{1/2} + C$

17. $\frac{1}{4}e^{4x-3} + C$

18. $\frac{1}{2}e^{x^2} + C$

19. $2e^{\sqrt{x}} + C$

20. $-\frac{1}{7}(e^{-x} + 2)^7 + C$

21. $1/4$

22. $8/3$

23. $26/3$

24. 0

25. $19/9$

26. $52/9$