How to find ABSOLUTE extrema of a function on a closed interval $[a, b]$.

STEP I: Find the critical number(s) of the function (say $x = c$) that are in the interval (a, b).

STEP II: Evaluate the **function** at the critical number(s). That is, calculate $f(c)$.

STEP III: Evaluate the **function** at each endpoint of $[a, b]$. That is, calculate $f(a)$ and $f(b)$.

STEP IV: The **absolute maximum** is the largest of the function values $f(c)$, $f(a)$, and $f(b)$.

STEP V: The **absolute minimum** is the smallest of the function values $f(c)$, $f(a)$, and $f(b)$.

EXAMPLE: Find the absolute extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

STEP I: $f'(x) = 12x^3 - 12x^2 = 12x^2(x - 1)$. So, the critical numbers are

$$12x^2(x - 1) = 0 \quad \Longrightarrow \quad x = 0 \ \text{ or } \ x = 1,$$

each of which lies in the interval $(-1, 2)$.

STEP II: Evaluate the function at these critical numbers:
$f(0) = 3(0)^4 - 4(0)^3 = 0$,
$f(1) = 3(1)^4 - 4(1)^5 = -1$.

STEP III: Evaluate the function at the endpoints of the interval $[-1, 2]$:
$f(-1) = 3(-1)^4 - 4(-1)^3 = 7$,
$f(2) = 3(2)^4 - 4(2)^3 = 16$.

STEP IV: The largest function value is $f(2) = 16$. Hence $x = 2$, $f(2) = 16$ is the absolute maximum of f on the interval $[-1, 2]$.

STEP V: The smallest function value is $f(1) = -1$. Hence $x = 1$, $f(1) = -1$ is the absolute minimum of f on the interval $[-1, 2]$.