Section 3.2: Curve Sketching Rational Functions

1. **DOMAIN**: Find all values of x for which $f(x)$ is defined.
2. **INTERCEPTS**:
   - x-intercepts: let $y = 0$, and solve for x
   - y-intercepts: let $x = 0$, and solve for y
3. **ASYMPTOTES**:
   - **Vertical asymptotes**: Find the values of a for which
     $$\lim_{x\to a} f(x) = \infty \qquad \text{or} \qquad \lim_{x\to a} f(x) = -\infty$$
     (NOTE: for a rational function, find where the denominator is equal to zero.)
   - **Horizontal asymptotes**: If $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$ then $y = L$ is a horizontal asymptote.
4. **INCREASING/DECREASING**:
   - Increasing when $f'(x) > 0$.
   - Decreasing when $f'(x) < 0$.
5. **RELATIVE MAX/MIN**:
   - Relative Max: $f(c)$ is a relative max if $f'(x)$ changes from + to - at $x = c$.
   - Relative Min: $f(c)$ is a relative max if $f'(x)$ changes from - to + at $x = c$.
6. **CONCAVITY**:
   - Concave up when $f''(x) > 0$
   - Concave down when $f''(x) < 0$
7. **POINTS OF INFLECTION**: P is a point of inflection if the concavity of f changes at P. (NOTE: To be a point of inflection P must be *in the domain* of f.)
8. **SKETCH GRAPH**

Example 1. Give a complete graph of

$$f(x) = \frac{x-4}{x-2}.$$

Be sure to find any horizontal and vertical asymptotes, show on a sign chart where the function is increasing/decreasing, concave up/concave down, and identifying (as ordered pairs) all relative extrema and inflection points. Also, identify the y-intercept.

Example 2. Give a complete graph of

$$f(x) = \frac{8}{x^2 - 4}.$$

Be sure to find any horizontal and vertical asymptotes, show on a sign chart where the function is increasing/decreasing, concave up/concave down, and identifying (as ordered pairs) all relative extrema and inflection points. Also, identify the y-intercept.

EXERCISES

Give a complete graph of the following functions. Be sure to find any horizontal and vertical asymptotes, show on a sign chart where the function is increasing/decreasing, concave up/concave down, and identifying (as ordered pairs) all relative extrema and inflection points. Also, identify the y-intercept.

1. $f(x) = \dfrac{2x+9}{x+3}$

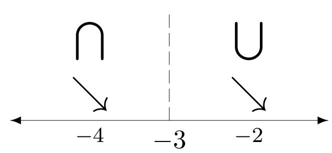
2. $f(x) = \dfrac{-2}{x+1}$

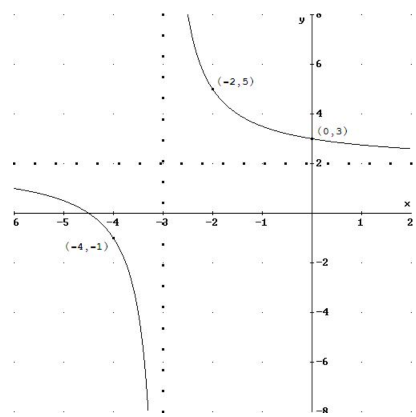
ANSWERS

1. Vertical asymptote at $x = -3$; horizontal asymptote at $y = 2$; no relative extrema or inflection points; y-intercept at $(0, 3)$.

$$f'(x) = \frac{-3}{(x+3)^2}$$

$$f''(x) = \frac{6}{(x+3)^3}$$





2. Vertical asymptote at $x = -1$; horizontal asymptote at $y = 0$ (i.e., the x-axis); no relative extrema or inflection points; y-intercept at $(0, -2)$.

$$f'(x) = \frac{2}{(x+1)^2}$$

$$f''(x) = \frac{-4}{(x+1)^3}$$