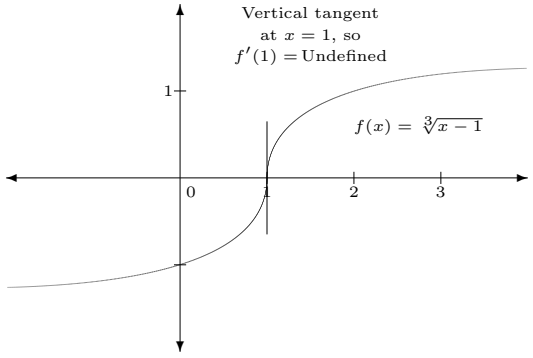
Section 2.7: Derivative Graphs

Recall. The derivative evaluated at $x = a$ is the slope of the tangent line to the graph of $y = f(x)$ at the point $\big(a, f(a)\big)$. Hence, to find $f'(a)$ from a graph, you must sketch the tangent line at the point $\big(a, f(a)\big)$ and then find the slope of that tangent line.

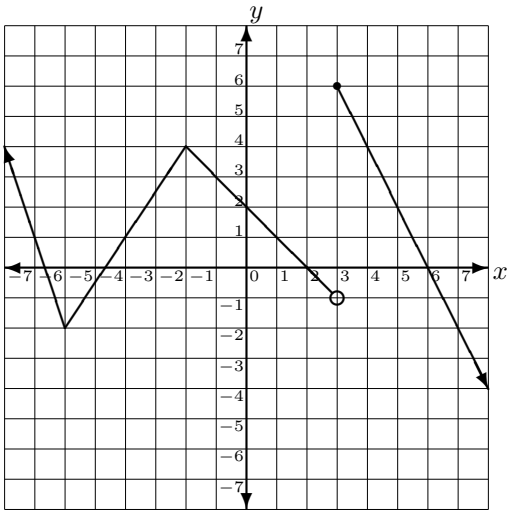
Result. If the graph of $y = f(x)$ has a sharp point at $x = a$, or if the function is not continuous at $x = a$ (i.e., the graph has a hole or jump at $x = a$), then the function will not have a derivative at $x = a$. In this case, we would say "f is **not differentiable** at $x = a$" and write "$f'(a) =$ Does not exist."

Note. The graph of a function can have a **vertical tangent** at $\big(a, f(a)\big)$. In this case, since the slope of a vertical line is undefined, the derivative at $x = a$ will also be undefined.

For example, the graph of $f(x) = \sqrt[3]{x-1}$ has a vertical tangent at $x = 1$. (See graph below.) Hence, $f'(1) =$ Undefined.



Example 1. For the function f graphed below, find the following:



1. $f'(-7) =$

2. $f'(-4) =$

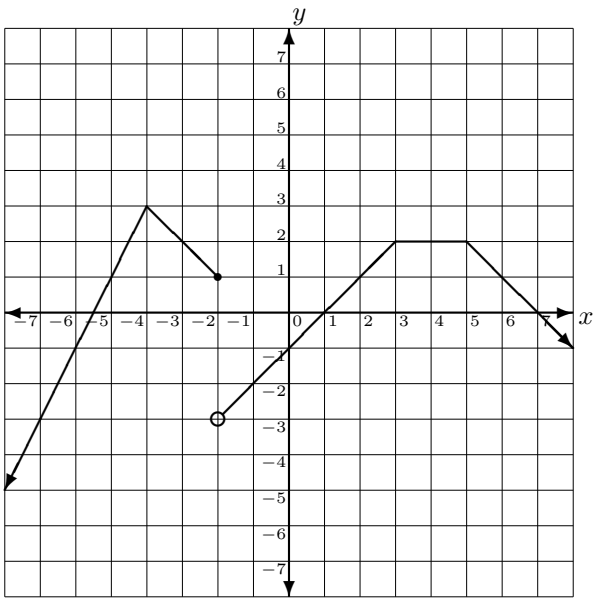
3. $f'(0) =$

4. $f'(5) =$

5. List the value(s) of x at which f is not differentiable.

Supplemental Exercises

For the function f graphed below, find the following:



1. $\lim_{x\to -4^-} f(x) =$
2. $\lim_{x\to -4^+} f(x) =$
3. $\lim_{x\to -4} f(x) =$
4. $f(-4) =$
5. $\lim_{x\to -2^-} f(x) =$
6. $\lim_{x\to -2^+} f(x) =$
7. $\lim_{x\to -2} f(x) =$
8. $f(-2) =$
9. $\lim_{x\to 0^-} f(x) =$
10. $\lim_{x\to 0^+} f(x) =$
11. $\lim_{x\to 0} f(x) =$
12. $f(0) =$
13. $\lim_{x\to 3^-} f(x) =$
14. $\lim_{x\to 3^+} f(x) =$
15. $\lim_{x\to 3} f(x) =$
16. $f(3) =$
17. $f'(-6) =$
18. $f'(-4) =$
19. $f'(-3) =$
20. $f'(-2) =$
21. $f'(0) =$
22. $f'(2) =$
23. $f'(3) =$
24. $f'(4) =$
25. $f'(5) =$
26. $f'(6) =$
27. List the value(s) of x at which f is discontinuous.
28. List the value(s) of x at which f is not differentiable.

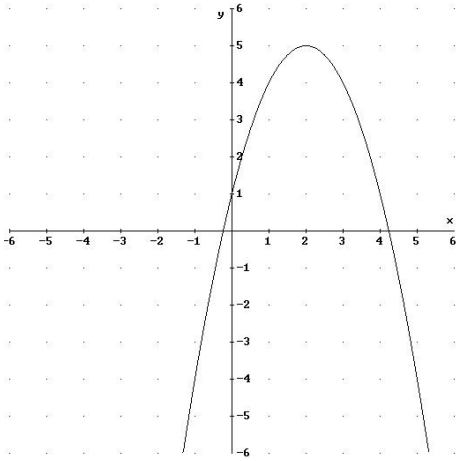
For the functions graphed below, find the following:

(a) the value(s) of x for which the graph has a horizontal tangent.

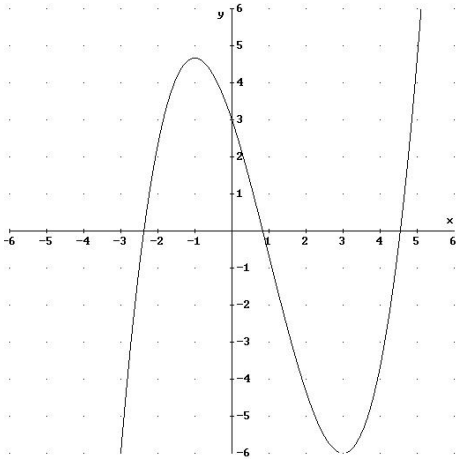
(b) the intervals where the derivative is positive.

(c) the intervals where the derivative is negative.

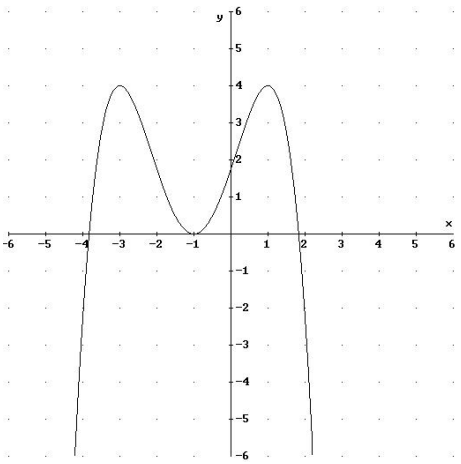
29.



30.



31.



ANSWERS

1. 3
2. 3
3. 3
4. 3
5. 1
6. -3
7. Does not exist
8. 1
9. -1
10. -1
11. -1
12. -1
13. 2
14. 2
15. 2
16. 2
17. 2
18. Does not exist
19. -1
20. Does not exist
21. 1
22. 1
23. Does not exist
24. 0
25. Does not exist
26. -1
27. $x = -2$
28. $x = -4, -2, 3, 5$

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29. (a) $x = 2$
    (b) $(-\infty, 2)$
    (c) $(2, \infty)$
30. (a) $x = -1, 3$
    (b) $(-\infty, -1) \cup (3, \infty)$
    (c) $(-1, 3)$
31. (a) $x = -3, -1, 1$
    (b) $(-\infty, -3) \cup (-1, 1)$
    (c) $(-3, -1) \cup (1, \infty)$