Section 4.3: Derivatives of Exponential and Logarithmic Functions

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THE DERIVATIVE OF $y = e^x$

$$\frac{d}{dx}[e^x] = e^x$$

and

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x).$$

Example 1. Find the derivative of $f(x) = 5e^{3x} + 4e^{-2x} + 7e^x + 8$.

Example 2. Find the derivative of $f(x) = 2xe^{5x}$.

Example 3. Find the derivative of $f(x) = e^{x^2}(7x - 3)^4$.

THE DERIVATIVE OF $y = \ln x$

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

and

$$\frac{d}{dx}\left[\ln\big(f(x)\big)\right] = \frac{1}{f(x)} \cdot f'(x).$$

Example 4. Find the derivative of $f(x) = 5\ln(x^2 + 7x + 1).$

Example 5. Find the derivative of $f(x) = (8x - 3)\ln(2x^2 + 5).$

Example 6. Find the derivative of $f(x) = e^{-x}\ln(x + 2).$

Supplemental Exercises

Find the derivative f'. Do not simplify.

1. $f(x) = (3e^x + 2e^{-x} + 5)(7x + 8)$

2. $f(x) = \dfrac{7e^x + 1}{3e^{-x} - 2}$

3. $f(x) = (e^{9x} + 3)\ln(4x^2 + 11)$

4. $f(x) = \dfrac{\ln(3x + 1)}{x - 7}$

5. $f(x) = [\ln(2x - 1)]^3$

ANSWERS

1. $f'(x) = [3e^x(1) + 2e^{-x}(-1)](7x + 8) + (3e^x + 2e^{-x} + 5)[7]$

2. $f'(x) = \dfrac{(3e^{-x} - 2)[7e^x(1)] - (7e^x + 1)[3e^{-x}(-1)]}{(3e^{-x} - 2)^2}$

3. $f'(x) = [e^{9x}(9)]\ln(4x^2 + 11) + (e^{9x} + 3)\left[\dfrac{1}{4x^2 + 11} \cdot (8x)\right]$

4. $f'(x) = \dfrac{(x - 7)\left[\dfrac{1}{3x + 1} \cdot (3)\right] - \ln(3x + 1)[1]}{(x - 7)^2}$

5. $f'(x) = 3[\ln(2x - 1)]^2 \cdot \left[\dfrac{1}{2x - 1} \cdot (2)\right]$