Section 2.1: Limits Graphically

Definition. We say that the **limit of** $f(x)$ **as** x **approaches** a **is equal to** L, written

$$\lim_{x \to a} f(x) = L,$$

if we can make the values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a, *but not equal to* a. In other words, as x approaches a (but never equaling a), $f(x)$ approaches L.

Definition. We say that the **limit of** $f(x)$ **as** x **approaches** a **from the left is equal to** L, written

$$\lim_{x \to a^-} f(x) = L,$$

if we can make the values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a, but *strictly less than* a (i.e., to the left of a as viewed on a number line). In other words, as x approaches a from the left (i.e., $x < a$), $f(x)$ approaches L.

Definition. We say that the **limit of** $f(x)$ **as** x **approaches** a **from the right is equal to** L, written

$$\lim_{x \to a^+} f(x) = L,$$

if we can make the values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a, but *strictly greater than* a (i.e., to the right of a as viewed on a number line). In other words, as x approaches a from the right (i.e., $a < x$), $f(x)$ approaches L.

Definition. Limits taken from the left or the right are called **one-sided limits**.

Result. If both one-sided limits equal L, then the two-sided limit must also equal L. Conversely, if the two-sided limit equals L, then both one-sided limits must also equal L. That is,

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L.$$

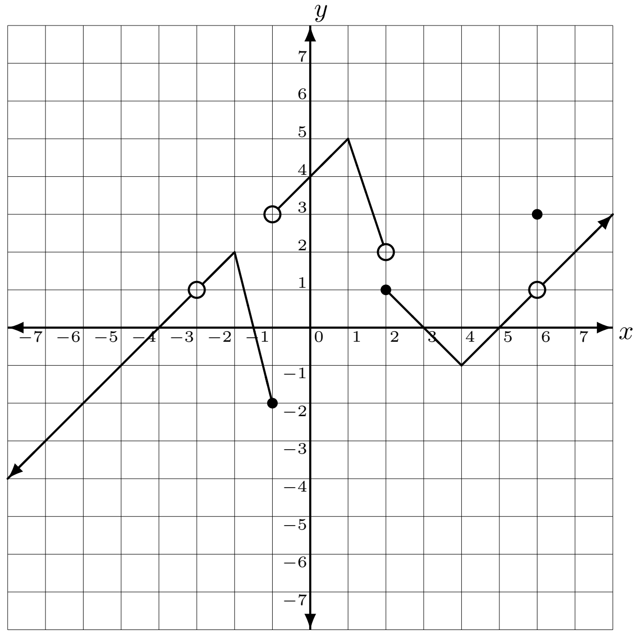
Definition. The function f is **continuous at** $x = a$ provided $f(a)$ is defined, $\lim_{x \to a} f(x)$ exists, and

$$\lim_{x \to a} f(x) = f(a).$$

In other words, the value of the limit equals the value of the function. Graphically, the function f is continuous at $x = a$ provided the graph of $y = f(x)$ does not have any holes, jumps, or breaks at $x = a$. (That is, the function is connected at $x = a$.)

If f is not continuous at $x = a$, then we say f is **discontinuous at** $x = a$ (or f has a **discontinuity at** $x = a$).

Example 1. For the function f graphed below, find the following:



1. $\lim_{x\to -3^-} f(x) =$

2. $\lim_{x\to -3^+} f(x) =$

3. $\lim_{x\to -3} f(x) =$

4. $f(-3) =$

5. $\lim_{x\to -1^-} f(x) =$

6. $\lim_{x\to -1^+} f(x) =$

7. $\lim_{x\to -1} f(x) =$

8. $f(-1) =$

9. $\lim_{x\to 2^-} f(x) =$

10. $\lim_{x\to 2^+} f(x) =$

11. $\lim_{x\to 2} f(x) =$

12. $f(2) =$

13. $\lim_{x\to 4^-} f(x) =$

14. $\lim_{x\to 4^+} f(x) =$

15. $\lim_{x\to 4} f(x) =$

16. $f(4) =$

17. $\lim_{x\to 6^-} f(x) =$

18. $\lim_{x\to 6^+} f(x) =$

19. $\lim_{x\to 6} f(x) =$

20. $f(6) =$

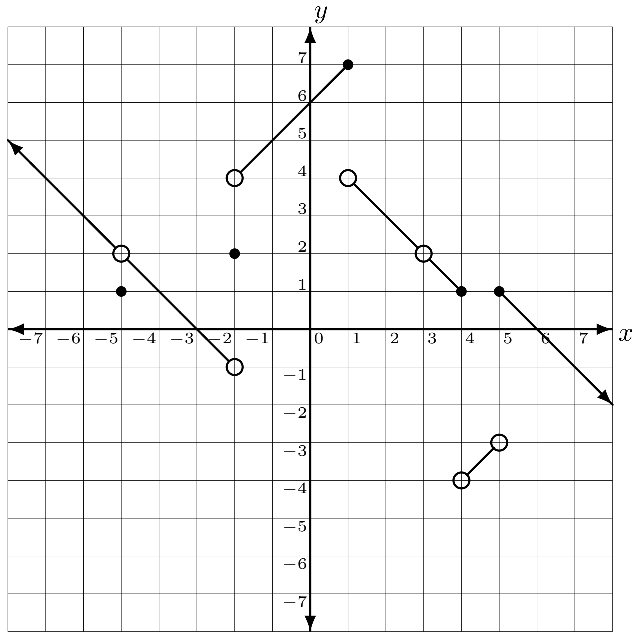
21. List the value(s) of x at which f is discontinuous.

Note that the function is continuous at $x = 4$ and hence

$$\lim_{x\to 4} f(x) = \lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4) = -1.$$

EXERCISES

For the function f graphed below, find the following:



1. $\lim_{x\to -5^-} f(x) =$
2. $\lim_{x\to -5^+} f(x) =$
3. $\lim_{x\to -5} f(x) =$
4. $f(-5) =$
5. $\lim_{x\to -2^-} f(x) =$
6. $\lim_{x\to -2^+} f(x) =$
7. $\lim_{x\to -2} f(x) =$
8. $f(-2) =$
9. $\lim_{x\to 0^-} f(x) =$
10. $\lim_{x\to 0^+} f(x) =$
11. $\lim_{x\to 0} f(x) =$
12. $f(0) =$
13. $\lim_{x\to 1^-} f(x) =$
14. $\lim_{x\to 1^+} f(x) =$
15. $\lim_{x\to 1} f(x) =$
16. $f(1) =$
17. $\lim_{x\to 3^-} f(x) =$
18. $\lim_{x\to 3^+} f(x) =$
19. $\lim_{x\to 3} f(x) =$
20. $f(3) =$
21. $\lim_{x\to 4^-} f(x) =$
22. $\lim_{x\to 4^+} f(x) =$
23. $\lim_{x\to 4} f(x) =$
24. $f(4) =$
25. $\lim_{x\to 5^-} f(x) =$
26. $\lim_{x\to 5^+} f(x) =$
27. $\lim_{x\to 5} f(x) =$
28. $f(5) =$
29. List the value(s) of x at which f is discontinuous.

ANSWERS

1. 2
2. 2
3. 2
4. 1
5. -1
6. 4
7. Does not exist
8. 2
9. 6
10. 6
11. 6
12. 6
13. 7
14. 4
15. Does not exist
16. 7
17. 2
18. 2
19. 2
20. Undefined
21. 1
22. -4
23. Does not exist
24. 1
25. -3
26. 1
27. Does not exist
28. 1
29. $x = -5, -2, 1, 3, 4, 5$