Section 2.3: Differentiation Forumlas

Reminder about notation. Let $y = f(x)$.

- The derivative may be denoted in the following ways:

$$f'(x) \qquad \text{OR} \qquad \frac{d}{dx}f(x) \qquad \text{OR} \qquad y' \qquad \text{OR} \qquad \frac{dy}{dx}.$$

- The derivative evaluated at a value of x, say for example $x = 5$, may be denoted in the following ways:

$$f'(5) \qquad \text{OR} \qquad \frac{d}{dx}f(x)\Big|_{x=5} \qquad \text{OR} \qquad y'\Big|_{x=5} \qquad \text{OR} \qquad \frac{dy}{dx}\Big|_{x=5}.$$

Derivative of a constant

For *any* constant c,

$$\frac{d}{dx}c = 0.$$

The Power Rule

For *any* constant exponent n,

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}.$$

Example 1. Find the derivative of the following functions:

(a) $f(x) = x^7$ (b) $g(x) = x^{5/7}$ (c) $h(x) = \sqrt[3]{x}$ (d) $k(x) = \dfrac{1}{x^5}$

Constant Multiple Rule

For *any* constant c,

$$\frac{d}{dx}\left[c \cdot f(x)\right] = c \cdot f'(x).$$

Example 2. Find the derivative of the following functions:

(a) $y = 4x^9$ (b) $f(x) = 3\sqrt{x}$ (c) $g(x) = \dfrac{3}{2x^4}$

The Sum/Difference Rule

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x).$$

Example 3. Find the derivative of

$$y = 3x^4 - 2x^3 + 4x^2 + \frac{2}{3x} - \frac{5}{4x^2} + 3\sqrt{x} - \frac{5}{6\sqrt[3]{x}} - 8x - 5.$$

Solution. First rewrite using positive and negative exponents:

$$y = 3x^4 - 2x^3 + 4x^2 + \frac{2}{3x} - \frac{5}{4x^2} + 3\sqrt{x} - \frac{5}{6\sqrt[3]{x}} - 8x - 5$$

$$y = 3x^4 - 2x^3 + 4x^2 + \frac{2}{3}x^{-1} - \frac{5}{4}x^{-2} + 3x^{1/2} - \frac{5}{6}x^{-1/3} - 8x - 5$$

$$y' = \boxed{12x^3 - 6x^2 + 8x - \frac{2}{3}x^{-2} + \frac{5}{2}x^{-3} + \frac{3}{2}x^{-1/2} + \frac{5}{18}x^{-4/3} - 8}$$

Example 4. Find the equation of the tangent line to $f(x) = 2x^3 + 4x^2 - 5x - 3$ at $x = -1$

Supplemental Exercises

Find the derivative y'. Do not simplify.

1. $y = 7x^4 - 6x^3 + 2x^2 + 9x - 1$
2. $y = -5x^4 + 2x^3 - 7x^2 + 4x - 3$
3. $y = -7x^{-4} + 2x^{-5} - 4x^{-1/2} + 8x^{2/3}$
4. $y = -3x^{-1} + 4x^{3/2} - 6x^{-1/2} + 5x^{-3}$
5. $y = 5\sqrt{x} + 6\sqrt[3]{x} - 4\sqrt[3]{x^2} + 2x - 1$
6. $y = 8\sqrt{x} - 2\sqrt[3]{x^2} - 15\sqrt[3]{x} + 9x - 2$
7. $y = \dfrac{8}{x} - \dfrac{2}{x^2} + \dfrac{7}{3x^3} - \dfrac{5}{2x^4} + 3$
8. $y = \dfrac{1}{3x} + \dfrac{5}{x^2} - \dfrac{8}{9x^3} + \dfrac{7}{4x^4} + 2$
9. $y = 5\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{7}{3\sqrt[3]{x}} + \dfrac{4}{5\sqrt[3]{x^2}}$
10. $y = \sqrt[3]{x} - \dfrac{1}{\sqrt[3]{x}} + \dfrac{1}{3\sqrt{x}} + \dfrac{5}{2\sqrt[3]{x^2}}$

ANSWERS

1. $y' = 28x^3 - 18x^2 + 4x + 9$

2. $y' = -20x^3 + 6x^2 - 14x + 4$

3. $y' = 28x^{-5} - 10x^{-6} + 2x^{-3/2} + \frac{16}{3}x^{-1/3}$

4. $y' = 3x^{-2} + 6x^{1/2} + 3x^{-3/2} - 15x^{-4}$

5. $y' = \frac{5}{2}x^{-1/2} + 2x^{-2/3} - \frac{8}{3}x^{-1/3} + 2$

6. $y' = 4x^{-1/2} - \frac{4}{3}x^{-1/3} - 5x^{-2/3} + 9$

7. $y' = -8x^{-2} + 4x^{-3} - 7x^{-4} + 10x^{-5}$

8. $y' = -\frac{1}{3}x^{-2} - 10x^{-3} + \frac{8}{3}x^{-4} - 7x^{-5}$

9. $y' = \frac{5}{2}x^{-1/2} - x^{-3/2} + \frac{7}{9}x^{-4/3} - \frac{8}{15}x^{-5/3}$

10. $y' = \frac{1}{3}x^{-2/3} + \frac{1}{3}x^{-4/3} - \frac{1}{6}x^{-3/2} - \frac{5}{3}x^{-5/3}$