
Section 2.4: Product and Quotient Rules

THE PRODUCT RULE

$$\frac{d}{dx} [f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x).$$

Example 1. Find the derivative of $y = (4x^3 - 3)(7x^2 - 4x + 5)$.

Example 2. Find the derivative of $y = (5x^2 - 2x + 1)(4x - 7)^3$.

Example 3. Find the derivative of $y = (5x + 3)\sqrt{2x - 1}$.

THE QUOTIENT RULE

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2} = \frac{\text{Bottom} \cdot \frac{d}{dx}(\text{Top}) - \text{Top} \cdot \frac{d}{dx}(\text{Bottom})}{(\text{Bottom})^2}.$$

Example 4. Find the derivative of $y = \frac{2x - 5}{x^2 - 7x + 2}$.

Example 5. Find the derivative of $y = \frac{x^2 + 9}{5x - 3}$.

Example 6. Find the derivative of $y = \frac{(4x + 3)^5}{x - 1}$.

Supplemental Exercises

Find the derivative f' . Do not simplify.

1. $f(x) = (3x - 7)(x^2 - 4x + 1)$

6. $f(x) = \frac{3x - 5}{7x + 1}$

2. $f(x) = (9x^2 + 1)(3x^2 + 4x - 3)$

7. $f(x) = \frac{8}{x^2 + 3x + 1}$

3. $f(x) = (7x + 1)(3x^2 - 4x - 9)^3$

8. $f(x) = \frac{4x - 5}{(8x - 3)^3}$

4. $f(x) = (5x - 7)\sqrt{4x - 5}$

9. $f(x) = \left(\frac{5x - 3}{x + 3}\right)^4$

5. $f(x) = (4x - 1)^2(5x + 2)^3$

10. $f(x) = \sqrt{\frac{x + 5}{2x + 3}}$

ANSWERS

1. $f'(x) = [3](x^2 - 4x + 1) + (3x - 7)[2x - 4]$

2. $f'(x) = [18x](3x^2 + 4x - 3) + (9x^2 + 1)[6x + 4]$

3. $f'(x) = [7](3x^2 - 4x - 9)^3 + (7x + 1) [3(3x^2 - 4x - 9)^2(6x - 4)]$

4. $f'(x) = [5](4x - 5)^{1/2} + (5x - 7) \left[\frac{1}{2}(4x - 5)^{-1/2}(4) \right]$

5. $f'(x) = [2(4x - 1)(4)](5x + 2)^3 + (4x - 1)^2 [3(5x + 2)^2(5)]$

6. $f'(x) = \frac{(7x + 1)[3] - (3x - 5)[7]}{(7x + 1)^2}$

7. $f'(x) = \frac{(x^2 + 3x + 1)[0] - 8[2x + 3]}{(x^2 + 3x + 1)^2}$

8. $f'(x) = \frac{(8x - 3)^3[4] - (4x - 5) [3(8x - 3)^2(8)]}{((8x - 3)^3)^2}$

9. $f'(x) = 4 \left(\frac{5x - 3}{x + 3} \right)^3 \left[\frac{(x + 3)[5] - (5x - 3)[1]}{(x + 3)^2} \right]$

10. $f'(x) = \frac{1}{2} \left(\frac{x + 5}{2x + 3} \right)^{-1/2} \left[\frac{(2x + 3)[1] - (x + 5)[2]}{(2x + 3)^2} \right]$