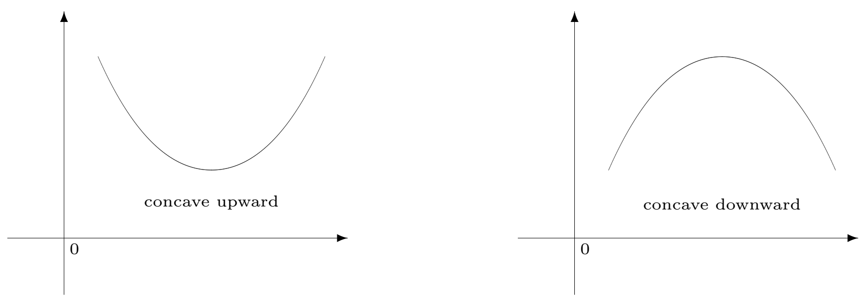
Section 3.2: Using the Second Derivative

Definition. If the graph of f lies above all of its tangents on an interval, then it is called **concave upward** $\left(\bigcup\right)$ on that interval. If the graph of f lies below all of its tangents on an interval, then it is called **concave downward** $\left(\bigcap\right)$ on that interval.



Concavity Results:

- If $f''(x) > 0$ on an interval, then the graph of f is concave upward $\left(\bigcup\right)$ on that interval.
- If $f''(x) < 0$ on an interval, then the graph of f is concave downward $\left(\bigcap\right)$ on that interval.

  NOTE: An **inflection point** is any point *in the domain* where the concavity changes. (f'' must be zero or undefined)

Example 1. Find the intervals where $f(x) = x^3 - 3x^2 - 9x + 7$ is concave up and the intervals where f is concave down. In addition, identify any points of inflection.

The Second Derivative Test. Let f be a function such that $f'(c) = 0$ and f'' exists on an open interval containing $x = c$.

- If $f''(c) > 0$, then f has a relative minimum at $x = c$.
- If $f''(c) < 0$, then f has a relative maximum at $x = c$.
- If $f''(c) = 0$, then no conclusion can be made about the point $\big(c, f(c)\big)$. Use the First Derivative Test.

Remark. Note that the Second Derivative Test requires, by definition, the calculation (and simplification) of the second derivative. For most functions, this can be a lengthy procedure. Also, as mentioned in item 3 above, the Second Derivative Test may be inconclusive when testing for relative extrema. This will never be the case when using the First Derivative Test.

Hence, as a rule of thumb, use the Second Derivative Test on those functions whose second derivative is easily calculated (e.g., polynomials). Otherwise, use the First Derivative Test. In fact, when finding relative extrema, most students choose to use the First Derivative Test exclusively.

Example 2. Use the Second Derivative Test to find the relative extrema of

$$f(x) = \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 - 12x + 1.$$