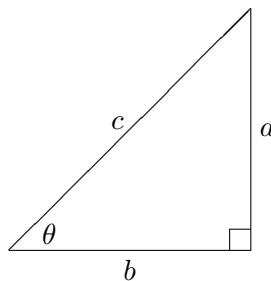


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# MATH 11022: Trigonometric Functions of Acute Angles

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**Definition.** For an arbitrary right triangle (*not necessarily on a coordinate system*), the six trigonometric functions of an angle  $\theta$  are defined as



$$\sin \theta = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

**MEMORIZE** The above definition can be memorized by the acronym “SohCahToa”:

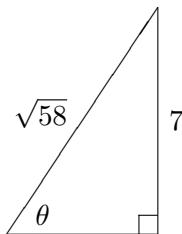
Sine = Opposite over Hypotenuse,

Cosine = Adjacent over Hypotenuse,

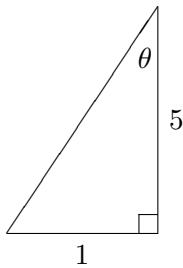
Tangent = Opposite over Adjacent.

(Some students write “ $S_{\text{h}}^{\text{o}} C_{\text{h}}^{\text{a}} T_{\text{a}}^{\text{o}}$ ” to emphasize the definition of sine, cosine, and tangent.)

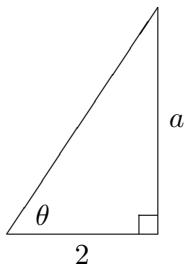
**Example 1:** Find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$ :



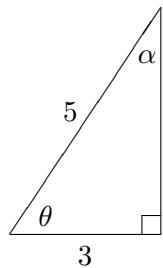
**Example 2:** Find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$ :



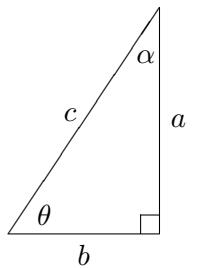
**Example 3:** Find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$ , written in terms of the variable  $a$ :



**Example 4:** Find the values of the six trigonometric functions of both  $\theta$  and  $\alpha$ :



## COFUNCTION IDENTITIES



$$\begin{array}{lll} \sin \theta = \cos \alpha & \tan \theta = \cot \alpha & \sec \theta = \csc \alpha \\ \cos \theta = \sin \alpha & \cot \theta = \tan \alpha & \csc \theta = \sec \alpha \end{array}$$

Alternatively,

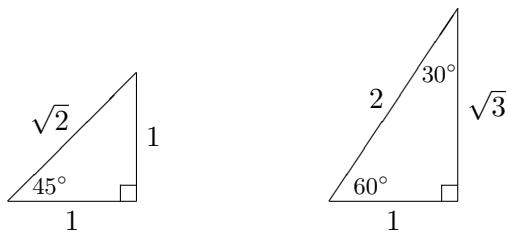
$$\begin{array}{lll} \sin \theta = \cos(90^\circ - \theta) & \tan \theta = \cot(90^\circ - \theta) & \sec \theta = \csc(90^\circ - \theta) \\ \cos \theta = \sin(90^\circ - \theta) & \cot \theta = \tan(90^\circ - \theta) & \csc \theta = \sec(90^\circ - \theta) \end{array}$$

**Example 5:** Find the exact value of  $\sin 45^\circ$ ,  $\cos 45^\circ$ , and  $\tan 45^\circ$ .

**Example 6:** Find the exact value of  $\sin 60^\circ$ ,  $\cos 60^\circ$ , and  $\tan 60^\circ$ .

**Example 7:** Find the exact value of  $\sin 30^\circ$ ,  $\cos 30^\circ$ , and  $\tan 30^\circ$ .

**IMPORTANT** *EFFECTIVE IMMEDIATELY*, you must know how to find the exact values of the six trig functions evaluated at  $\theta = 30^\circ$ ,  $60^\circ$ , and  $45^\circ$ . Use the following triangles and “SohCahToa.”



**Example 8:** Find the exact value of the following:

$$(a) \quad 3 \tan 60^\circ + 4 \sin 30^\circ =$$

$$(b) \quad \sec 30^\circ \sec 60^\circ + \csc 30^\circ \csc 60^\circ =$$

$$(c) \quad \frac{\sin 60^\circ - \cos 60^\circ}{\cos 30^\circ} =$$

## SUPPLEMENTAL EXERCISES

Simplify the following.

$$1. \quad \frac{\sin 60^\circ + \tan 45^\circ}{\cos 60^\circ} =$$

$$2. \quad \frac{1 - \cos 30^\circ}{1 + \cos 30^\circ} =$$

$$3. \quad \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$$

$$4. \quad \frac{\tan 45^\circ - \sin 45^\circ}{\sin 45^\circ + \cos 45^\circ} =$$

$$5. \quad \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} =$$

## SOLUTIONS

$$1. \quad \frac{\sin 60^\circ + \tan 45^\circ}{\cos 60^\circ} =$$

SOLUTION.

$$\frac{\sin 60^\circ + \tan 45^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2} + 1}{\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2} + \frac{2}{2}}{\frac{1}{2}} = \frac{\frac{\sqrt{3} + 2}{2}}{\frac{1}{2}} = \frac{\sqrt{3} + 2}{2} \cdot \frac{2}{1} = \boxed{\sqrt{3} + 2}$$

$$2. \quad \frac{1 - \cos 30^\circ}{1 + \cos 30^\circ} =$$

SOLUTION.

$$\frac{1 - \cos 30^\circ}{1 + \cos 30^\circ} = \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}} = \frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{\frac{2}{2} + \frac{\sqrt{3}}{2}} = \frac{\frac{2 - \sqrt{3}}{2}}{\frac{2 + \sqrt{3}}{2}} = \frac{2 - \sqrt{3}}{2} \cdot \frac{2}{2 + \sqrt{3}} = \boxed{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}}$$

$$3. \quad \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$$

SOLUTION.

$$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \left( \frac{1}{\sqrt{3}} \right)}{1 - \left( \frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \cdot \frac{3}{2} = \boxed{\frac{3}{\sqrt{3}}}$$

$$4. \quad \frac{\tan 45^\circ - \sin 45^\circ}{\sin 45^\circ + \cos 45^\circ} =$$

SOLUTION.

$$\frac{\tan 45^\circ - \sin 45^\circ}{\sin 45^\circ + \cos 45^\circ} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{2}}} = \frac{\frac{\sqrt{2} - 1}{\sqrt{2}}}{\frac{2}{\sqrt{2}}} = \frac{\sqrt{2} - 1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2} - 1}{2}}$$

5. SOLUTION.

$$\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1) \left( \frac{1}{\sqrt{3}} \right)}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3} + 1}{\sqrt{3}}}{\frac{\sqrt{3} - 1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3} - 1}$$

$$= \boxed{\frac{\sqrt{3} + 1}{\sqrt{3} - 1}}$$