MATH 11022: Angles

Definitions:

• An **angle** is a pair of rays having the same endpoint or **vertex**. An angle can also be formed by rotating a ray around its endpoint.

- Angles are drawn by showing the direction and amount of rotation from the **initial side** to the **terminal side**.
- If the rotation is counterclockwise, the angle is **positive**; if the rotation is clockwise, the angle is **negative**.
- Angles are usually denoted by lower case Greek letters:

$$\label{eq:alpha} \begin{split} \alpha &= \text{``alpha''} \quad \beta &= \text{``beta''} \quad \gamma &= \text{``gamma''} \quad \theta &= \text{``theta''} \quad \phi &= \text{``phi''} \\ \text{Upper case English letters are also used to denote angles.} \end{split}$$

- An acute angle is an angle measuring more than 0° but less than 90° .
- A **right angle** is an angle measuring 90° .
- An obtuse angle is an angle measuring more than 90° but less than 180°.
- A straight angle is an angle measuring exactly 180°.
- Two angles are **complementary angles** if the sum of their measures is 90°.
- Two angles are **supplementary angles** if the sum of their measures is 180°.

Degree Measure

- 360° = one revolution (measured counterclockwise)
- $1^{\circ} = \frac{1}{360}$ revolution



Example 1: Convert $30^{\circ}6'27''$ into decimal degrees.

Example 2: Convert 58°12′18″ into decimal degrees.

Example 3: Convert 25.139° into DMS.

Definition. An angle θ is in **standard position** if its vertex is at the origin on a rectangular coordinate system and its initial side is the along the positive *x*-axis.

Definition. If the terminal side of θ lies on the x or y-axes, then θ is a quadrantal angle.

MEMORIZE You must memorize the quadrantal angles:



Example 4: Draw each angle:

(a)
$$\alpha = 45^{\circ}$$
 (d) $\theta = 480^{\circ}$

(b)
$$\beta = -45^{\circ}$$
 (e) $\phi = -225^{\circ}$

(c)
$$\gamma = 230^{\circ}$$
 (f) $C = -310^{\circ}$

Definition. Angles that have the same initial and terminal sides are coterminal angles.

Result. Let θ be any angle. Then there are an infinite number of angles that are coterminal with θ . These coterminal angles all differ by a multiple of 360°.

Example 5: Give two positive and two negative angles that are coterminal with

(a) $\theta = 40^{\circ}$

(b)
$$\theta = 160^{\circ}$$

(c) $\theta = -100^{\circ}$