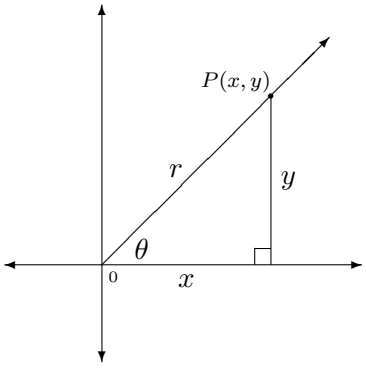
MATH 11022: Definition of Trigonometric Functions

Definition of the Trigonometric Functions: Let θ be an angle in standard position and let $P(x, y)$ be any point (other than the origin) on the terminal side of θ. Let r be the distance from the origin to the point P. (i.e., $r = \sqrt{x^2 + y^2}$.) Then the six trigonometric functions of θ are



"*sine*"	$\sin\theta = \dfrac{y}{r}$		$\csc\theta = \dfrac{r}{y}$	"*cosecant*"
"*cosine*"	$\cos\theta = \dfrac{x}{r}$		$\sec\theta = \dfrac{r}{x}$	"*secant*"
"*tangent*"	$\tan\theta = \dfrac{y}{x}$		$\cot\theta = \dfrac{x}{y}$	"*cotangent*"

MEMORIZE The above definition must be committed to memory.

NOTES:

- When the terminal side of θ lies on y-axis, the x-coordinate is zero. Therefore, $\tan\theta$ and $\sec\theta$ are undefined.
- When the terminal side of θ lies on x-axis, the y-coordinate is zero. Therefore, $\cot\theta$ and $\csc\theta$ are undefined.

Example 1: Find the value of the six trigonometric functions of the angle θ if P is a point on the terminal side of θ.

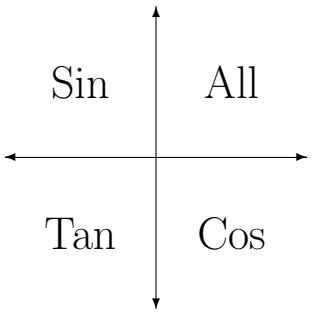
(a) $P = (-4, -3)$

(b) $P = (-\sqrt{5}, 3)$

Quadrant Signs of the Trigonometric Functions

- If the terminal side of θ lies in Quadrant I, then all six trigonometric functions of θ are positive.
- If the terminal side of θ lies in Quadrant II, then only $\sin\theta$ and $\csc\theta$ are positive.
- If the terminal side of θ lies in Quadrant III, then only $\tan\theta$ and $\cot\theta$ are positive.
- If the terminal side of θ lies in Quadrant IV, then only $\cos\theta$ and $\sec\theta$ are positive.

MEMORIZE The above result must be committed to memory.



"AllSinTanCos"

Example 2: Without using a calculator, determine if the following are positive or negative:

(a) $\sin 100^\circ$

(b) $\cos 220^\circ$

(c) $\tan 340^\circ$

(d) $\sin(-210^\circ)$

(e) $\cos(-130^\circ)$

(f) $\tan(-330^\circ)$

Example 3:

(a) If $\sin\theta$ is negative and $\tan\theta$ is negative, then θ lies in which quadrant(s)?

(b) If $\cos\theta$ is negative and $\sin\theta$ is positive, then θ lies in which quadrant(s)?

(c) If $\csc\theta$ is negative and $\cot\theta$ is positive, then θ lies in which quadrant(s)?

(d) If $\sin\theta$ is positive, then θ lies in which quadrant(s)?

(e) If $\tan\theta$ is negative, then θ lies in which quadrant(s)?

Example 4: If $\sin\theta = \dfrac{3}{7}$ and $\cos\theta$ is negative, find the value of the other five trigonometric functions.

Example 5: If $\cos\theta = \frac{2}{3}$ and $\tan\theta$ is negative, find the value of the other five trigonometric functions.

Example 6: If $\tan\theta = -\frac{2}{9}$, find the value of $\sin\theta$ and $\cos\theta$.