MATH 11022: Half Angle Identities

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x}$$

$$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x}$$

NOTE: The $\pm$ sign is determined by the quadrant in which the angle $\frac{x}{2}$ terminates.

Example 1: Find the exact value of

(a) $\sin 22.5^\circ =$

(b) $\cos 157.5^\circ =$

(c) $\cot \frac{\beta}{2}$, given $\tan \beta = -\frac{\sqrt{5}}{2}$ with $90^\circ < \beta < 180^\circ$.