MATH 11022: Basic Trigonometric Properties and Identities

COFUNCTION IDENTITIES

$$\sin\theta = \cos(90^\circ - \theta) \qquad \tan\theta = \cot(90^\circ - \theta) \qquad \sec\theta = \csc(90^\circ - \theta)$$

$$\cos\theta = \sin(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

RANGES OF THE TRIGONOMETRIC FUNCTIONS

- For any angle θ, $\quad -1 \leq \sin\theta \leq 1$.
- For any angle θ, $\quad -1 \leq \cos\theta \leq 1$.
- $-\infty < \tan\theta < \infty$.
- $-\infty < \cot\theta < \infty$.
- $\sec\theta \leq -1 \quad$ or $\quad \sec\theta \geq 1$. $\qquad$ That is, $|\sec\theta| \geq 1$
- $\csc\theta \leq -1 \quad$ or $\quad \csc\theta \geq 1$. $\qquad$ That is, $|\csc\theta| \geq 1$

RECIPROCAL IDENTITIES:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

QUOTIENT IDENTITIES:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

PYTHAGOREAN IDENTITIES (Memorize for Exam 2.)

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sin^2\theta = 1 - \cos^2\theta \qquad \tan^2\theta = \sec^2\theta - 1 \qquad \cot^2\theta = \csc^2\theta - 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

QUADRANT SIGNS OF THE TRIGONOMETRIC FUNCTIONS:

- If the terminal side of θ lies in Quadrant I, then all six trigonometric functions of θ are positive.
- If the terminal side of θ lies in Quadrant II, then only $\sin\theta$ and $\csc\theta$ are positive.
- If the terminal side of θ lies in Quadrant III, then only $\tan\theta$ and $\cot\theta$ are positive.
- If the terminal side of θ lies in Quadrant IV, then only $\cos\theta$ and $\sec\theta$ are positive.