MATH 11022: Inverse Trig Functions

Definition. The **inverse sine function**, denoted $\sin^{-1}$, is defined by

$$\sin^{-1} x = \theta \quad \longleftarrow \text{ is the same as } \longrightarrow \quad \sin\theta = x,$$

where $-1 \le x \le 1$ and $-90° \le \theta \le 90°$. That is, θ is an angle in the 1st or 4th quadrant.

The inverse sine function is also called the **arcsine function** and is denoted arcsin.

Example 1: Find the exact value of

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

(b) $\arcsin\left(\frac{1}{2}\right) =$

(c) $\arcsin\left(-\frac{1}{\sqrt{2}}\right) =$

(d) $\arcsin\left(-\frac{1}{2}\right) =$

(e) $\sin^{-1}(1) =$

(f) $\sin^{-1}(0) =$

Example 2: On a calculator, find the following (accurate to two decimal places):

(a) $\sin^{-1}(0.3) =$

(b) $\arcsin(-0.97) =$

(c) $\arcsin(0.197) =$

(d) $\sin^{-1}(2.5) =$

Definition. The **inverse cosine function**, denoted $\cos^{-1}$, is defined by

$$\cos^{-1} x = \theta \quad \longleftarrow \text{ is the same as } \longrightarrow \quad \cos\theta = x,$$

where $-1 \leq x \leq 1$ and $0° \leq \theta \leq 180°$. That is, $\underline{\theta \text{ is an angle in the 1st or 2nd quadrant}}$.

The inverse cosine function is also called the **arccosine function** and is denoted arccos.

Example 3: Find the exact value of

(a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

(d) $\arccos\left(-\frac{\sqrt{3}}{2}\right) =$

(b) $\arccos\left(-\frac{1}{\sqrt{2}}\right) =$

(e) $\cos^{-1}(0) =$

(c) $\arccos\left(-\frac{1}{2}\right) =$

(f) $\cos^{-1}(-1) =$

Example 4: On a calculator, find the following (accurate to two decimal places):

(a) $\cos^{-1}(0.53) =$

(c) $\arccos(-0.8) =$

(b) $\arccos(0.178) =$

(d) $\cos^{-1}(3.6) =$

Definition. The **inverse tangent function**, denoted $\tan^{-1}$, is defined by

$$\tan^{-1} x = \theta \quad \longleftarrow \text{ is the same as } \longrightarrow \quad \tan\theta = x,$$

where $-\infty < x < \infty$ and $-90° < \theta < 90°$. That is, $\underline{\theta \text{ is an angle in the 1st or 4th quadrant}}$.

The inverse tangent function is also called the **arctangent function** and is denoted arctan.

Example 5: Find the exact value of

(a) $\tan^{-1}(\sqrt{3}) =$

(d) $\arctan(-1) =$

(b) $\arctan\left(-\frac{1}{\sqrt{3}}\right) =$

(e) $\tan^{-1}(-\sqrt{3}) =$

(c) $\tan^{-1}(1) =$

(f) $\arctan(0) =$

Example 6: On a calculator, find the following (accurate to two decimal places):

(a) $\tan^{-1}(0.82) =$

(c) $\tan^{-1}(1000) =$

(b) $\arctan(-3.7) =$

(d) $\tan^{-1}(100,000) =$

Example 7: Find the exact value of

(a) $\sin\left[\arccos\left(\frac{1}{2}\right)\right] =$

(b) $\tan\left[\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right] =$

(c) $\cos\left[\arctan(-1)\right] =$

(d) $\sec\left[\arcsin\left(\frac{1}{2}\right)\right] =$

(e) $\cot\left[\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right] =$

(f) $\sin\left[\tan^{-1}\left(\sqrt{3}\right)\right] =$

Example 8: Find the exact value of

(a) $\sin\left[\arctan\left(-\frac{4}{3}\right)\right] =$

(b) $\sec\left[\sin^{-1}\left(\frac{1}{5}\right)\right] =$

(c) $\tan\left[\arccos\left(-\frac{2}{5}\right)\right] =$

(d) $\sin\left[\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(-\frac{4}{5}\right)\right]=$

(e) $\cos\big[2\arctan(3)\big]=$