## MATH 11022: Solving Oblique Triangles

Result. Keep in mind these properties from geometry:

- The sum of the angles of a triangle is $180^{\circ}$.
- The sum of the two shorter sides of a triangle is always greater than the longest side.

- In a triangle, the largest side is opposite the largest angle and the smallest side is opposite the smallest angle.

- If an angle is not opposite the largest side, then it must be acute. However, the angle opposite the largest side may be acute or obtuse. Hence, when inspecting a triangle prior to solving it, you can only determine if an angle must be acute; you can never determine if an angle must be obtuse. This result is very handy when using inverse sine $\left(\sin ^{-1}\right)$ to find an angle.

Example 1: Determine which angles (if any) must be acute and state why. (Note that you can not determine if an angle is obtuse simply by inspection.)


## General Notes:

- Use the Law of Sines when the initial condition is AAS (or SAA), ASA, or ASS. Note that ASS is the ambiguous case so you must be careful when using inverse sine $\left(\sin ^{-1}\right)$.
- Use the Law of Cosines when the initial condition is SAS or SSS. The Law of Cosines is the safest way to find an angle.
- To avoid round-off error, take any intermediate calculations to at least four decimal places.
- If possible, always try to use the original information given in the problem. Avoid using any values that were calculated in a previous step.
- Grading Note. Whenever you use inverse sine $\left(\sin ^{-1}\right)$ to find an acute angle, you MUST EXPLAIN WHY the angle is acute and not obtuse. (See the last result on the previous page.) If you fail to do this, you will not receive full credit for the problem. Note that the use of inverse cosine $\left(\cos ^{-1}\right)$ does not require this explanation; inverse cosine will always yield the proper angle.

Example 2: Solve the following triangles. Give answers to two decimal places. Note that all figures are not drawn to scale.
(a)
Answer: $95^{\circ}, 2.58,2.84$

(b)

(c) Answer: $30.75^{\circ}, 24.15^{\circ}, 125.10^{\circ}$

(d)

Answer: $79.11^{\circ}, 40.90^{\circ}, \quad 5.29$

(e)

Answer: $33.26^{\circ}, 99.74^{\circ}, 5.39$

(f)

Answer: No triangle can be formed

(g) $\quad$ Answer: $45.83^{\circ}, 108.17^{\circ}, 23.84$, or $134.17^{\circ}, 19.83^{\circ}, 8.51$


Example 3: Solve for $x$ and $y$. Give answers to two decimal places. Answer: $x=2.91, y=4.15$


Example 4: Solve for $x$ and $h$. Give answers to two decimal places. Answer: $x=11.05, \quad h=9.27$


Example 5: Solve for $x$. Give answer to two decimal places.
Answer: $x=230.94$


Example 6: Solve for $x$. Give answer to two decimal places.
Answer: $x=98.15$
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Example 7: Solve for $x$. Give answer to two decimal places.
Answer: $x=63.70$


Example 8: Solve for $\theta$. Give answer to two decimal places. $\quad$ Answer: $\theta=37.89^{\circ}$


