## MATH 11022: Polar Coordinates

Definition. This is the polar coordinate system:



Any point P in the plane can be represented by an ordered pair  $P = (r, \theta)$ , called the **polar** coordinates of P, where r is the distance from O to P and  $\theta$  is the angle between the polar axis and the segment  $\overline{OP}$ .

## Notes:

- If r is negative, then  $(r, \theta)$  is defined to be the point that lies |r| units from the pole in the direction opposite to that given by  $\theta$ .
- Points in the plane do not have a unique polar representation. For example,  $(2, 30^{\circ})$ ,  $(2, 390^{\circ})$ ,  $(2, -330^{\circ})$ , and  $(-2, 210^{\circ})$  all represent the same point. In general, any point  $(r, \theta)$  can also be represented by

$$(r, \theta + 360^{\circ}n)$$
 and  $(-r, \theta + 180^{\circ}(2n+1)),$ 

for any integer n. In particular, note that  $(-r, \theta)$  and  $(r, \theta + 180^{\circ})$  represent the same point.

• The pole O is represented by the polar coordinates  $(0, \theta)$ , for any angle  $\theta$ .

## **Example 1:** Plot the following points:



$$A = (2, 60^{\circ}) \qquad \qquad G = (-4, 150^{\circ})$$

 $B = (2, 420^{\circ}) \qquad \qquad H = (-3, -90^{\circ})$ 

$$C = (2, -60^{\circ}) \qquad \qquad I = (-4, -45^{\circ})$$

 $D = (-2, 60^{\circ}) \qquad \qquad J = (0, 135^{\circ})$ 

$$E = (2, 240^{\circ})$$
  $K = (4, 15^{\circ})$ 

$$F = (3, 135^{\circ}) \qquad \qquad L = (-3, 165^{\circ})$$

RELATIONSHIP BETWEEN POLAR AND RECTANGULAR COORDINATES • To change from polar to rectangular coordinates, use the formulas  $x = r \cos \theta$  and  $y = r \sin \theta$ • To change from rectangular to polar coordinates, use the formulas 1.  $r = \sqrt{x^2 + y^2}$ 2.  $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ , if x > 0, 3.  $\theta = \tan^{-1}\left(\frac{y}{x}\right) + 180^\circ$ , if x < 0.

**Example 2:** Find the rectangular coordinates of the following points. Give answers to two decimal places.

(a)  $(3, 120^{\circ})$ 

(b)  $(2, 230^{\circ})$ 

**Example 3:** Find polar coordinates for the following points. Give  $\theta$  as a positive angle, accurate to two decimal places.

(a)  $(1,\sqrt{3})$ 

(b) 
$$(-2, -2)$$

(c) (4, -1)

(d) (-3,2)