## MATH 11022: Polar Coordinates

Definition. This is the polar coordinate system:


Any point $P$ in the plane can be represented by an ordered pair $P=(r, \theta)$, called the polar coordinates of $P$, where $r$ is the distance from $O$ to $P$ and $\theta$ is the angle between the polar axis and the segment $\overline{O P}$.

## Notes:

- If $r$ is negative, then $(r, \theta)$ is defined to be the point that lies $|r|$ units from the pole in the direction opposite to that given by $\theta$.
- Points in the plane do not have a unique polar representation. For example, $\left(2,30^{\circ}\right)$, $\left(2,390^{\circ}\right),\left(2,-330^{\circ}\right)$, and $\left(-2,210^{\circ}\right)$ all represent the same point. In general, any point $(r, \theta)$ can also be represented by

$$
\left(r, \theta+360^{\circ} n\right) \quad \text { and } \quad\left(-r, \theta+180^{\circ}(2 n+1)\right),
$$

for any integer $n$. In particular, note that $(-r, \theta)$ and $\left(r, \theta+180^{\circ}\right)$ represent the same point.

- The pole $O$ is represented by the polar coordinates $(0, \theta)$, for any angle $\theta$.

Example 1: Plot the following points:

$A=\left(2,60^{\circ}\right)$
$B=\left(2,420^{\circ}\right)$

$$
C=\left(2,-60^{\circ}\right)
$$

$$
I=\left(-4,-45^{\circ}\right)
$$

$$
D=\left(-2,60^{\circ}\right)
$$

$$
J=\left(0,135^{\circ}\right)
$$

$$
E=\left(2,240^{\circ}\right)
$$

$$
K=\left(4,15^{\circ}\right)
$$

$$
F=\left(3,135^{\circ}\right)
$$

$$
L=\left(-3,165^{\circ}\right)
$$

## Relationship Between Polar and Rectangular Coordinates

- To change from polar to rectangular coordinates, use the formulas

$$
x=r \cos \theta \quad \text { and } \quad y=r \sin \theta
$$

- To change from rectangular to polar coordinates, use the formulas

1. $r=\sqrt{x^{2}+y^{2}}$
2. $\quad \theta=\tan ^{-1}\left(\frac{y}{x}\right)$, if $x>0$,
3. $\quad \theta=\tan ^{-1}\left(\frac{y}{x}\right)+180^{\circ}$, if $x<0$.

Example 2: Find the rectangular coordinates of the following points. Give answers to two decimal places.
(a) $\left(3,120^{\circ}\right)$
(b) $\left(2,230^{\circ}\right)$

Example 3: Find polar coordinates for the following points. Give $\theta$ as a positive angle, accurate to two decimal places.
(a) $(1, \sqrt{3})$
(b) $(-2,-2)$
(c) $(4,-1)$
(d) $(-3,2)$

