
MATH 11022: Radians

Definition: An angle with its vertex at the center of a circle (called a **central angle**) that intercepts an arc on the circle in length to the radius of the circle has a measure of **one radian**.

RADIANS–DEGREES

$$\pi \text{ rad} = 180^\circ \quad \text{and} \quad 2\pi \text{ rad} = 360^\circ$$

CONVERSION FACTORS

- To convert from radians into degrees, multiply by $\frac{180^\circ}{\pi}$.
- To convert from degrees into radians, multiply by $\frac{\pi}{180^\circ}$.

Example 1: Convert to degrees:

(a) $\frac{5\pi}{4}$

(b) $\frac{3\pi}{2}$

(c) $\frac{\pi}{6}$

(d) $\frac{4\pi}{3}$

Example 2: Convert to radians:

(a) 60°

(b) 90°

(c) -45°

(d) 240°

NOTES:

- If a given angle has no units, then it is assumed to be measured in radians. That is, radian measure is unitless.
- Radian measure amounts to measuring angles in terms of real numbers which is necessary in Calculus.
- Radian measure allows us to treat the trigonometric functions as functions with domains of real numbers rather than angles.
- It is **NOT** true that all angles given in radians must contain π .
- NOTATION: Angles denoted by the variable x are always given in radians. Angles denoted by a Greek letter are usually given in degrees.

Example 3: Find the exact value of

(a) $\sin \frac{5\pi}{6} =$

(b) $\cos \frac{5\pi}{3} =$

(c) $\tan \left(-\frac{11\pi}{6} \right) =$

(d) $\sin \pi =$

ARC LENGTH

The length s of the arc intercepted on a circle of radius r by a central angle θ , *measured in radians*, is

$$s = r\theta$$

Example 4: Find the length of the arc intercepted by a central angle of 3.6 rad on a circle of radius 24 cm.

Example 5: What is the measure (in radians) of a central angle which intercepts an arc of 13 cm on a circle of radius 5 cm?

AREA OF A SECTOR

The area of a sector of a circle of radius r and central angle θ , *measured in radians*, is

$$A = \frac{1}{2}r^2\theta$$

Example. For a circle of radius 10 cm, find the length of the arc intercepted by a central angle of 2.3 rad and then find the area of this sector.