MATH 11022: Graphs of Sine and Cosine

Definition. A function f is **periodic** if there is a positive number p such that $f(x+p) = f(x)$. The smallest such positive number is the **period** of f. If f has period p, then one **cycle** of f is the graph of f over any interval of length p.

Periodic Properties of the Trigonometric Functions

The functions sine, cosecant, cosine, and secant have period 2π (or 360°). That is,

$$\sin(x + 2\pi) = \sin x \quad \text{or} \quad \sin(\theta + 360^\circ) = \sin\theta$$
$$\cos(x + 2\pi) = \cos x \quad \text{or} \quad \cos(\theta + 360^\circ) = \cos\theta$$
$$\csc(x + 2\pi) = \csc x \quad \text{or} \quad \csc(\theta + 360^\circ) = \csc\theta$$
$$\sec(x + 2\pi) = \sec x \quad \text{or} \quad \sec(\theta + 360^\circ) = \sec\theta$$

The functions tangent and cotangent have period π (or 180°). That is,

$$\tan(x + \pi) = \tan x \quad \text{or} \quad \tan(\theta + 180^\circ) = \tan\theta$$
$$\cot(x + \pi) = \cot x \quad \text{or} \quad \cot(\theta + 180^\circ) = \cot\theta$$

Example 1: Graph $y = \sin x$

Example 2: Graph $y = \cos x$

Results:

- One cycle is from $0 \leq x \leq 2\pi$.
- The domains of $f(x) = \sin x$ and $g(x) = \cos x$ are all real numbers $\mathbb{R}$.
- For all x, $\boxed{-1 \leq \sin x \leq 1}$ and $\boxed{-1 \leq \cos x \leq 1}$

Example 3: Graph one cycle of

(a) $y = \sin x + 2$

(b) $y = \cos x - 1$

(c) $y = -\sin x$

(d) $y = -\cos x$

Definition. For the functions

$$y = a\sin x \qquad \text{and} \qquad y = a\cos x$$

the number $|a|$ is called the **amplitude** of the graph and is one half the difference between the curve's maximum and minimum values.

Example 4: Graph one cycle of

(a) $y = 3\sin x$

(b) $y = -2\cos x$

Result. The sine and cosine curves

$$y = a\sin(kx) \qquad \text{and} \qquad y = a\cos(kx)$$

have amplitude $|a|$ and period $\frac{2\pi}{k}$.

Example 5: Find the amplitude and period of the following functions and sketch the graph of one cycle.

(a) $y = 5\cos\left(\frac{\pi}{2}x\right)$

(b) $y = -2\sin\left(\frac{1}{4}x\right)$

(c) $y = -3\cos(\pi x)$

(d) $y = 7\sin(4x)$

Result. The sine and cosine curves

$$y = a\sin[k(x-\phi)] \qquad \text{and} \qquad y = a\cos[k(x-\phi)]$$

have amplitude $|a|$, period $\frac{2\pi}{k}$, and **phase shift** ϕ. The angle ϕ is also called the **phase constant** or **phase angle**.

Definition. The graph of $y = a\sin[k(x-b)]$ is called a **sinusoid**.

Example 6: Find the amplitude, period, and phase shift of the following functions and sketch the graph of one cycle.

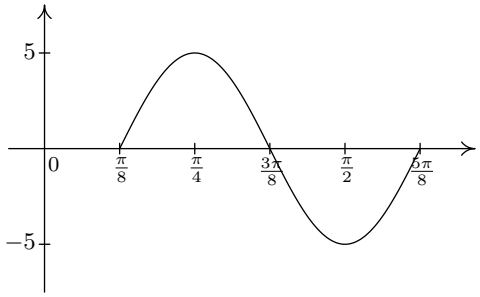
(a) $y = 2\sin(4x - \pi)$

(b) $y = -3\cos\left(2x + \frac{\pi}{2}\right)$

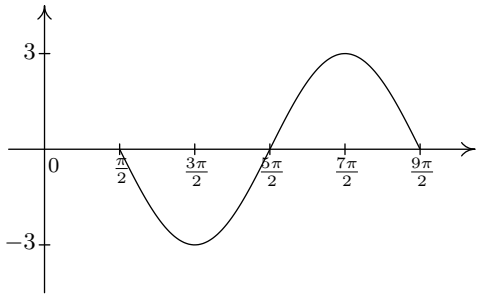
(c) $y = 4\cos\left(\frac{\pi}{2}x + \pi\right)$

Example 7: Find the amplitude, period, phase shift, and equation of the following sinusoids.

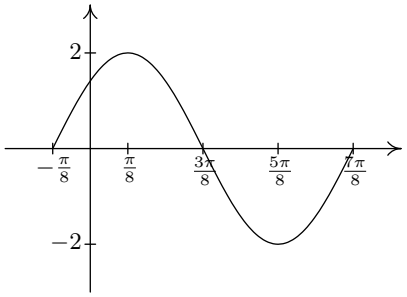
(a)



(b)



(c)



(d)