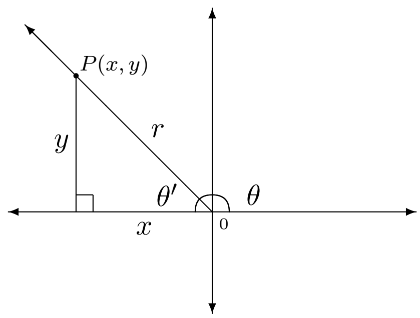
MATH 11022: Trigonometric Functions of Non-Acute Angles

Definitions: Let θ be a nonquadrantal angle in standard position with point (x, y) on its terminal side. Then the right triangle formed by dropping a perpendicular line segment from (x, y) to the $x-$axis is called the **reference triangle**. The **reference angle** for θ, denoted θ', is the acute angle formed between the terminal side of θ and the $x-$axis.



Example 1: Find the reference angle θ' for the following:

(a) $\theta = 150°$

(b) $\theta = 220°$

(c) $\theta = 300°$

(d) $\theta = -135°$

(e) $\theta = -315°$

(f) $\theta = -236°$

The Reduction Principle

The trigonometric functions of any nonquadrantal angle θ are equal to those of the reference angle θ' associated with θ, except possibly for the sign (positive or negative). The sign can be determined by considering the quadrant in which the terminal side of θ lies.

Example 2: Find the exact value of the following:

(a) $\sin 225°$

(b) $\cos 300°$

(c) $\tan 120°$

(d) $\sin(-225°)$

(e) $\cos(-150°)$

(f) $\cot 330°$

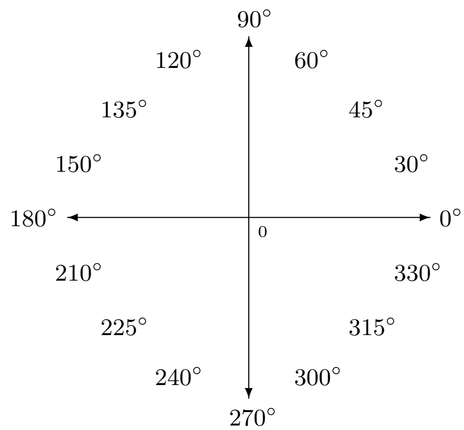
(g) $\sec(-30°)$

(h) $\csc 225°$

(i) $\sin 270°$

(j) $\cos 135°$

IMPORTANT *EFFECTIVE IMMEDIATELY*, you must be able to quickly (and correctly) find the **exact** trigonometric values of the following angles:



Coterminal Angles

For any integer n,

$$\sin(\theta + 360°n) = \sin\theta$$

$$\cos(\theta + 360°n) = \cos\theta$$

$$\tan(\theta + 360°n) = \tan\theta$$

Example 3: Find the exact value of

(a) $\sin 420° =$

(b) $\cos 840° =$

(c) $\tan(-675°) =$

Example 4: Find all angles, $0^\circ \leq \theta < 360^\circ$ for which

(a) $\sin\theta = \frac{1}{2}$

(d) $\tan\theta = -1$

(b) $\cos\theta = \frac{\sqrt{2}}{2}$

(e) $\cos\theta = 0$

(c) $\sin\theta = -\frac{\sqrt{3}}{2}$

(f) $\sin\theta = -\frac{1}{\sqrt{2}}$