## MATH 11022: Vector Addition by Components

Definition. A vector $\mathbf{A}$ is in standard position if its initial point is at the origin on a rectangular coordinate system. The angle $\theta, 0^{\circ} \leq \theta<360^{\circ}$, that $\mathbf{A}$ makes with the positive $x$-axis is called the standard position angle of the vector $\mathbf{A}$.

Definition. Let $\mathbf{A}$ be a vector in standard position and let $\theta$ be its standard position angle. Then the $x$ and $y$-components of $\mathbf{A}$, denoted $A_{x}$ and $A_{y}$ respectively, are defined to be

$$
A_{x}=\|\mathbf{A}\| \cos \theta \quad \text { and } \quad A_{y}=\|\mathbf{A}\| \sin \theta
$$

Note that the $x$ and $y$-components of $\mathbf{A}$ are scalars, not vectors. Finding these components is called resolving the vector into its $x$ and $y$-components.

Example 1: Find the $x$ and $y$-components of the following vectors. Give answers to two decimal places. Note that all figures are not drawn to scale.
(a)

(b)

(c)

(d)


Vector Addition by Components
To add vectors $\mathbf{A}$ and $\mathbf{B}$ by components (and hence find the resultant vector $\mathbf{R}$ ), perform the following steps:

1. Place $\mathbf{A}$ and $\mathbf{B}$ in standard position.
2. Resolve $\mathbf{A}$ and $\mathbf{B}$ into their $x$ and $y$-components. That is, find $A_{x}$, $A_{y}, B_{x}$, and $B_{y}$.
3. The $x$ and $y$-components of the resultant vector $\mathbf{R}$ are

$$
R_{x}=A_{x}+B_{x} \quad \text { and } \quad R_{y}=A_{y}+B_{y}
$$

4. Sketch $\mathbf{R}$ using $R_{x}$ and $R_{y}$.
5. The magnitude of $\mathbf{R}$ is

$$
\|\mathbf{R}\|=\sqrt{R_{x}^{2}+R_{y}^{2}}
$$

The standard position angle of $\mathbf{R}$ is

$$
\theta=\tan ^{-1}\left(\frac{R_{y}}{R_{x}}\right),
$$

appropriately modified to fit the quadrant in which $\mathbf{R}$ lies in.

Example 3: Find the magnitude and standard position angle of the resultant vector $\mathbf{R}$. Give final answers to two decimal places. To avoid round-off error, take any intermediate calculations to at least four decimal places. Note that all figures are not drawn to scale.
(a)

(b)

(c)

(d)


