MATH 11022: Vectors

Definition. A **vector** is a quantity that has both magnitude and direction.

Definition. A **scalar** is a quantity that can be completely specified by a single number and unit, and therefore have only magnitude and not direction.

Definition. A **geometric vector in the plane** is a quantity that possesses both a length and a direction and can be represented by a directed line segment.

Definition. The **magnitude** (or **length**, or **norm**) of the vector $\mathbf{A}$, denoted $\|\mathbf{A}\|$ (or $|\mathbf{A}|$), is the length of the directed line segment.

Definition. Two vectors are **equal** if they have the same magnitude and direction. Thus, a vector may be translated from one location to another as long as the magnitude and direction are not changed.

Definition. The vector $-\mathbf{A}$ has the same magnitude as $\mathbf{A}$, but has the opposite direction as $\mathbf{A}$.

Definition. For any number n, the vector $n\mathbf{A}$ is called a **scalar multiple of A**. If n is positive, then the vector $n\mathbf{A}$ has the same direction as $\mathbf{A}$, but has magnitude $n\|\mathbf{A}\|$. If n is negative, then the vector $n\mathbf{A}$ has the opposite direction as $\mathbf{A}$ and has magnitude $|n|\|\mathbf{A}\|$. If n is zero, then $n\mathbf{A}$ is the **zero vector 0** and has magnitude 0 and is assigned no direction.

Result. The sum of two vectors $\mathbf{A}$ and $\mathbf{B}$ can be found using the **tail-to-tip rule**: translate $\mathbf{B}$ so that its tail end is at the tip end of $\mathbf{A}$. Then the vector from the tail end of $\mathbf{A}$ to the tip end of $\mathbf{B}$ is called the **resultant vector** $\mathbf{R}$ and is the vector sum of $\mathbf{A}$ and $\mathbf{B}$. That is, $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

Note. The sum of two (nonparallel) vectors $\mathbf{A}$ and $\mathbf{B}$ can also be found using the **parallelogram rule**: the sum of (nonparallel) vectors $\mathbf{A}$ and $\mathbf{B}$ is the diagonal of the parallelogram formed using $\mathbf{A}$ and $\mathbf{B}$ as adjacent sides.