MATH 11022: Verifying Identities

Definitions:

- An **equation** is a statement that two expressions are equal.
- An **identity** is an equation that is true for all values of the variables in the domain of the expressions.
- An equation that is not an identity is a **conditional equation**.

RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \iff \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \iff \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \iff \tan\theta = \frac{1}{\cot\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sin^2\theta = 1 - \cos^2\theta \qquad \tan^2\theta = \sec^2\theta - 1 \qquad \cot^2\theta = \csc^2\theta - 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

QUOTIENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

NEGATIVE ANGLE IDENTITIES

$$\sin(-\theta) = -\sin\theta \quad \text{and} \quad \cos(-\theta) = \cos\theta \quad \text{and} \quad \tan(-\theta) = -\tan\theta$$

SUGGESTIONS FOR VERIFYING TRIGONOMETRIC IDENTITIES

- Know the fundamental identities.
- Attempt to transform the more complicated side into the other side.
- Keep in mind all of the tricks from algebra: factoring, FOIL, common denominators, breaking apart numerators, etc.
- When stuck, write all trig functions in terms of $\sin\theta$ and $\cos\theta$ and see what happens.
- If you are really stuck and the expression contains $1+\sin\theta$, $1-\sin\theta$, $1+\cos\theta$, or $1-\cos\theta$, consider multiplying by a conjugate (but not always).
- The answer is always in front of you—work towards it.

Examples: Verify the following identities.

1. $\sin\theta\cot\theta = \cos\theta$

2. $(1 + \sin\theta)(1 - \sin\theta) = \dfrac{1}{\sec^2\theta}$

3. $\sec\theta - \cos\theta = \sin\theta\tan\theta$

4. $\dfrac{\sec^2\theta}{1+\cot^2\theta} = \tan^2\theta$

5. $\dfrac{\cos\theta + \tan\theta}{\sin\theta\cos\theta} = \csc\theta + \sec^2\theta$

6. $$\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$$

7. $$\frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$$

8. $\dfrac{1+2\sin\theta+\sin^2\theta}{\cos^2\theta} = \dfrac{1+\sin\theta}{1-\sin\theta}$

IMPORTANT Avoid these common errors:

- $\csc\theta+\sec\theta \neq \dfrac{1}{\sin\theta+\cos\theta}$
- $\dfrac{\overbrace{\sec\theta}^{\curvearrowleft}+\sin\theta}{\sin\theta} \neq \dfrac{\sin\theta}{\cos\theta\sin\theta}$; You can not move $\sec\theta$ because of the addition.
- $\dfrac{\cancel{\sin\theta\cos\theta}+1}{\cancel{\sin\theta\cos\theta}+\cos\theta} \neq \dfrac{1}{\cos\theta}$; You can not cancel because of the addition.