The 1st Derivative Test

The 1st Derivative Test (page 241)

Let $x = c$ be a critical number for a function that is continuous on an open interval (a, b) containing c. If f is differentiable on (a, b), except possibly at $x = c$, then $f(c)$ can be classified as follows:

1. *If $f'(x)$ changes sign from negative to positive at $x = c$, then the point $(c, f(c))$ is a relative minimum of f.*
2. *If $f'(x)$ changes sign from positive to negative at $x = c$, then the point $(c, f(c))$ is a relative maximum of f.*
3. *If $f'(x)$ does not change sign from positive to negative (or vice versa) at $x = c$, then f does not have a relative max or min at the point $(c, f(c))$.*

How to find relative extrema using the 1st Derivative Test

STEP I: Find the points $x = c$ in the domain of f where $f'(c) = 0$ or $f'(c)$ does not exist, i.e. the critical numbers of f.

STEP II: Plot these points on a number line.

STEP III: Determine the sign of f' both to the left and right of these critical numbers by evaluating f' at "test points." Keep in mind that if $f' > 0$, then the function is increasing ($\nearrow$), and if $f' < 0$, then the function is decreasing ($\searrow$).

STEP IV:

(a) If f' changes from $-$ to $+$ at $x = c$ (i.e. $\searrow$ to $\nearrow$), then $f(c)$ is a **relative minimum**.

(b) If f' changes from $+$ to $-$ at $x = c$ (i.e. $\nearrow$ to $\searrow$), then $f(c)$ is a **relative maximum**.

(c) If f' does not change sign at $x = c$ (i.e. $\searrow$ to $\searrow$ OR $\nearrow$ to $\nearrow$), then $f(c)$ is neither a relative max or min.