# The 2nd Derivative Test

**The 2nd Derivative Test** (page 245)

*Let $f$ be a function such that $f'(c) = 0$ and $f''$ exists on an open interval containing $x = c$.*

1. *If $f''(c) > 0$, then the point $(c, f(c))$ is a relative minimum,*
2. *if $f''(c) < 0$, then the point $(c, f(c))$ is a relative maximum,*
3. *if $f''(c) = 0$, then no conclusion can be made about the point $(c, f(c))$. Use the 1st Derivative Test to determine if $x = c$ is a relative max or min.*

### How to find relative extrema using the 2nd Derivative Test

**STEP I:** Find the numbers $x = c$ where $f'(c) = 0$.
(Note: If there is a point $x = c$ in the domain of $f$ where $f'(c)$ does **not** exist, then you must test this point by the 1st Derivative Test.)

**STEP II:** Calculate $f''(x)$.

**STEP III:** (a) If $f''(c) > 0$, then $f(c)$ is a **relative minimum**.
(b) If $f''(c) < 0$, then $f(c)$ is a **relative maximum**.
(c) If $f''(c) = 0$, then no conclusion can be made about this number. Use the 1st Derivative Test.

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**EXAMPLE:** Find the relative extrema of $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x + 1$.

**STEP I:** $f'(x) = x^2 + x - 12 = (x + 4)(x - 3)$. So,

$$(x + 4)(x - 3) = 0 \quad \Longrightarrow \quad x = -4 \text{ or } x = 3.$$

**STEP II:** $f''(x) = 2x + 1$.

**STEP III:** $f''(-4) = 2(-4) + 1 = -7 < 0$ (concave down $\bigcap$), so a **relative maximum** occurs at $x = -4$. The relative maximum is

$$f(-4) = \frac{1}{3}(-4)^3 + \frac{1}{2}(-4)^2 - 12(-4) + 1 = 107/3.$$

Also, $f''(3) = 2(3) + 1 = 7 > 0$ (concave up $\bigcup$), so a **relative minimum** occurs at $x = 3$. The relative minimum is

$$f(3) = \frac{1}{3}(3)^3 + \frac{1}{2}(3)^2 - 12(3) + 1 = -43/2.$$