MATH 12002 CHAPTER 2 EXAM SPRING 2002

1. Evaluate the limit, if it exists.

   (a) $\displaystyle\lim_{x\to -2}\left[\frac{1}{x+2}+\frac{4}{x^2-4}\right]=$

   (b) $\displaystyle\lim_{x\to 0}\frac{(x-4)^2-16}{x}=$

   (c) $\displaystyle\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=$

   (d) $\displaystyle\lim_{x\to 3}\frac{2x^2-5x-3}{x^2+2x-15}=$

   (e) $\displaystyle\lim_{x\to 1}\frac{x^2+x-2}{x+7}=$

   (f) $\displaystyle\lim_{x\to 3}\frac{4-\sqrt{x^2+7}}{x-3}=$

   (g) $\displaystyle\lim_{x\to 3^-}\frac{|x-3|}{x-3}=$

2. Use the Intermediate Value Theorem to show that there is a root of the equation

$$x^5+2x^2-x+2=\sqrt{x^2+8}$$

   on the interval $(0,1)$. Be specific.

3. Determine if the following functions are continuous or discontinuous at the given point a. If it is discontinuous at a, state which condition fails.

   (a) $f(x)=\begin{cases} 7x-5 & \text{if } x\ge 3 \\ 2x & \text{if } x<3 \end{cases}\qquad a=3$

   (b) $g(x)=\begin{cases} x^3+2 & \text{if } x\le -1 \\ x^2+x+1 & \text{if } x>-1 \end{cases}\qquad a=-1$

   (c) $h(x)=\dfrac{2x^2+3x-5}{4x^2-x-3}\qquad a=1$

4. Find the equation of the tangent line to the curve $y=2x^2-3x$ at the point $(3,9)$.

5. Locate the discontinuities for $f(x)=\dfrac{2}{1+\cos x}$

6. Given a graph of f, be able to find limits, function values, points at which f is discontinuous, points at which f is continuous from the left or from the right.

ANSWERS

1. (a) $-\frac{1}{4}$

   (b) -8

   (c) 4

   (d) $\frac{7}{8}$

   (e) 0

   (f) $-\frac{3}{4}$

   (g) -1

2. Let $f(x) = x^5 + 2x^2 - x + 2 - \sqrt{x^2 + 8}$. Now, f is continuous on $[0, 1]$ and $f(0) = 2 - \sqrt{8} < 0$ whereas $f(1) = 1 > 0$. Therefore, by the Intermediate Value Theorem, there is a root $c \in (0, 1)$ such that $f(c) = 0$.

3. (a) f is discontinuous at $x = 3$ since the $\lim_{x \to 3} f(x) = \text{dne}$.

   (b) g is continuous at $x = -1$.

   (c) h is discontinuous at $x = 1$ since h is not defined at $x = 1$.

4. $y = 9x - 18$

5. f is discontinuous whenever $1 + \cos x = 0$. Therefore, f is discontinuous at $x = \pi + 2k\pi$ where k is any integer.

6. See instructor for details.