1. Find the absolute maximum and absolute minimum of $f(x) = 3x^5 - 5x^3 - 1$ on the interval $[-1, 2]$.

2. Find the interval(s) on which $f(x) = x - 2\sin x,\ 0 \le x \le 2\pi$, is decreasing.

3. True or False

   (a) If $f'(c) = 0$, then f has a local maximum or local minimum at $x = c$.

   (b) If $f'(x) = g'(x)$ for all x then $f(x) = g(x)$.

   (c) If f is differentiable on the open interval (a, b), and $f(c)$ is a local maximum for f in (a, b), then $f'(c) = 0$.

   (d) If $f'(c) = 0$ and $f''(c) < 0$ then f has a local minimum at c.

   (e) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of the curve $f(x)$.

4. Evaluate the following limits. (For infinite limits, determine if the answer is ∞ or $-\infty$.)

   (a) $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+3x} - x\right) =$

   (b) $\displaystyle\lim_{x\to-\infty}\frac{-x^3+2x^2+1}{5x^3-7} =$

   (c) $\displaystyle\lim_{x\to\infty}\left(x^2 - x^4\right) =$

5. Find the critical values for $f(x) = 2x^3 + 3x^2 - 6x + 4$

6. Determine the local maximum(s) and local minimum(s) for $f(x) = 2x^3 - 3x^2 - 12x$.

7. A company has a cost function

$$C(x) = \frac{x^2}{2} + 12x + 1200$$

   and demand function $p(x) = 100 - \dfrac{x}{2}$. How many units should it make to maximize its profit?

8. A rectangular region with area 3200 square feet is to be enclosed within a fence. The two sides which run north-south will use fencing materials costing \$1.00 per foot, while the other two sides require fencing materials which cost \$2.00 per foot. Find the dimensions of the region which minimize material costs.

9. Let $f(x) = \dfrac{2(x^2+x+1)}{3(x+2)^2}$, so that

$$f'(x) = \frac{2x}{(x+2)^3} \quad \text{and} \quad f''(x) = \frac{-4(x-1)}{(x+2)^4}.$$

   (a) Find the domain

   (b) Calculate the $y-$intercept of f.

   (c) Calculate the horizontal asymptote(s), if it exists.

   (d) Calculate the vertical asymptote(s), if it exists.

   (e) Determine where f is increasing and where f is decreasing. Label answers.

   (f) Find all local extrema of f.

   (g) Determine where f is concave up and where f is concave down.

   (h) Find all points of inflection.

   (i) Sketch the graph of f, clearly indicating all of the information obtained above.

10. Given the graph of the derivative be able to identify

   (a) Determine the interval(s) where f is increasing.

   (b) Determine the interval(s) where f is decreasing.

   (c) Find the x values of all local maxima.

   (d) Find the x values of all local minima.

   (e) Determine the interval(s) where f is concave up.

   (f) Determine the interval(s) where f is concave down.

ANSWERS

1. absolute min $= -3$ and absolute max $= 55$
2. $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$
3. (a) F
   (b) F
   (c) T
   (d) F
   (e) F
4. (a) $\frac{3}{2}$
   (b) $-\frac{1}{5}$
   (c) $-\infty$
5. $x = \frac{-1 \pm \sqrt{5}}{2}$
6. min$= (2, -20)$; max $= (-1, 7)$
7. 44 units
8. 80 feet (N-S) by 40 feet (E-W)
9. (a) $x \neq -2$
   (b) $\left(0, \frac{1}{6}\right)$
   (c) $y = \frac{2}{3}$
   (d) $x = -2$
   (e) increasing: $(-\infty, -2) \cup (0, \infty)$; decreasing $(-2, 0)$
   (f) local minimum$= \left(0, \frac{1}{6}\right)$
   (g) concave up $(-\infty, -2) \cup (-2, 1)$; concave down $(1, \infty)$
   (h) $\left(1, \frac{2}{9}\right)$
   (i) See instructor.
10. See instructor.