MATH 12002 EXAM #1 Spring 2005

1. Evaluate the limit, if it exists.

   (a) $\displaystyle\lim_{x\to 2}\frac{3x^2-5x-2}{2x^2+x-10}=$

   (b) $\displaystyle\lim_{x\to 2}\frac{3x+2}{\sqrt{3-x}+1}=$

   (c) $\displaystyle\lim_{x\to 2}\frac{5-\sqrt{8x+9}}{x-2}=$

   (d) $\displaystyle\lim_{x\to 3^-}\frac{2x}{x-3}=$

   (e) $\displaystyle\lim_{x\to 1}\frac{16-(x-5)^2}{x-1}=$

2. Use the Intermediate Value Theorem to show that there is a root of the equation

$$\sqrt{3x+2}+x^3=3x+7$$

   Be specific.

3. Determine if the following functions are continuous or discontinuous at the given point a. If it is discontinuous at a, state which condition fails.

   (a) $f(x)=\begin{cases}\dfrac{x+1}{x-2} & \text{if } x\geq 3\\ x^2-2 & \text{if } x<3\end{cases}\qquad a=3$

   (b) $g(x)=\begin{cases}\dfrac{x^2-4}{x+2} & \text{if } x\neq -2\\ 4 & \text{if } x=-2\end{cases}\qquad a=-2$

4. Locate the discontinuities for

$$f(x)=\frac{3}{\sqrt{3}+2\cos 2x}$$

5. Let $f(x)=2-6x-3x^2$.

   (a) Find $f'(x)$ using the definition of the derivative.

   (b) Find the slope of the tangent line to f at $x=1$.

   (c) Find the equation of the tangent line in part (b).

6. Consider the function $f(x)=\begin{cases}\sin x & \text{if } x\leq \frac{3\pi}{4}\\ \cos x & \text{if } x>\frac{3\pi}{4}\end{cases}$

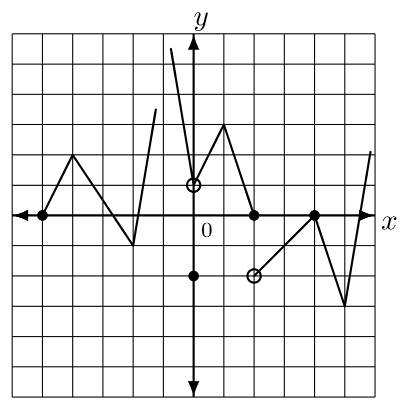
   Find

   (a) $\displaystyle\lim_{x\to \frac{3\pi}{4}^+} f(x)=$

   (b) $\displaystyle\lim_{x\to \frac{3\pi}{4}^-} f(x)=$

   (c) $\displaystyle\lim_{x\to \frac{3\pi}{4}} f(x)=$

7. The graph of f is given below. Find



   (a) $f(0)=$

   (b) $f'(-3)=$

   (c) $f'(\frac{3}{2})=$

   (d) $\displaystyle\lim_{x\to -1} f(x)=$

   (e) $\displaystyle\lim_{x\to 2} f(x)=$

   (f) determine the value(s) of x for which f is discontinuous.

   (g) For each of the value(s) in (g), determine if f is continuous from the left or continuous from the right.

   (h) determine the value(s) of x for which f is not differentiable.

   (i) determine the value(s) of x for which $f'(x)=0$

8. Given the graph of f sketch the graph of f'

ANSWERS

1. (a) $\frac{7}{9}$
   (b) 4
   (c) $-\frac{4}{5}$
   (d) $-\infty$
   (e) 8

2. f is continuous on its domain of $[-2/3, \infty)$; $f(0) = \sqrt{2} - 7 < 0$ and $f(3) = \sqrt{11} + 27 - 9 - 7 > 0$. Therefore, by IVT, there is a constant $c \in (0, 3)$ such that $f(c) = 0$

3. (a) f discontinuous at $x = 3$ since $\lim_{x \to 3} f(x) =$dne
   (b) f discontinuous at $x = -2$ since $g(-2) \neq \lim_{x \to -2} g(x)$

4. $x = \dfrac{5\pi}{12} + n\pi \quad x = \dfrac{7\pi}{12} + n\pi$ where n is any integer

5. (a) $-6 - 6x$
   (b) $m = -12$
   (c) $y = -12x + 5$

6. (a) $-\dfrac{\sqrt{2}}{2}$
   (b) $\dfrac{\sqrt{2}}{2}$
   (c) dne

7. (a) -2
   (b) $-\dfrac{3}{2}$
   (c) -3
   (d) ∞
   (e) dne
   (f) $-1, 0, 2$
   (g) 2 is continuous from the left
   (h) $x = -4, -2, -1, 0, 1, 2, 4$
   (i) $x = 5$

8. see instructor