MATH 12002 EXAM #3 Spring 2005

1. Evaluate the following limits. (For infinite limits, determine if the answer is ∞ or $-\infty$.) Show all work. (NO SHORTCUTS!!!)

   (a) $\lim_{x\to-\infty} \left(x^3 - x^5\right) =$

   (b) $\lim_{x\to\infty} \dfrac{7x^4 - 5x^2 + 3}{3x^4 - 5x^3 + 6x} =$

   (c) $\lim_{x\to-\infty} \dfrac{\sqrt{8x^2 + 7x}}{3 - 4x} =$

2. Find the critical values for $f(x) = 4x^{5/2} + 2x^{3/2} - 6x^{1/2}$

3. Determine the local maximum(s) and local minimum(s) for $f(x) = 3x^{2/3} - x$.

4. Find the absolute maximum and absolute minimum of $f(x) = 3x^4 - 16x^3 + 18x^2$ on the interval $[-1, 4]$.

5. Find the interval(s) on which $f(x) = 2\cos x + \sin 2x,\ 0 \le x \le 2\pi$, is increasing and where f is decreasing.

6. Determine the intervals where $f(x) = 4\cos x - x^2$ is concave up and where f is concave down.

7. An appliance firm is marketing a new refrigerator. It determines that in order to sell x refrigerators, the price per refrigerator must be $p(x) = 280 - 0.4x$. It also determines that the total cost of producing x refrigerators is given by $C(x) = 5000 + 0.6x^2$. How many refrigerators must the company sell in order to maximize profit?

8. A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container. (Round answer to two decimal places.)

9. Sketch the graph of the function f that satisfies the given conditions:

   $f(1) = f'(2) = 0 \qquad \lim_{x\to 0} f(x) = \infty$

   $\lim_{x\to-\infty} f(x) = 3 \qquad \lim_{x\to\infty} f(x) = 3$

   $f'(x) > 0$ for $0 < x < 2$

   $f'(x) < 0$ for $x < 0$ and $x > 2$

   $f''(x) < 0$ for $x < 0$ and $0 < x < 3$

   $f''(x) > 0$ for $x > 3$

10. Given the graph of the derivative, f', answer the following questions about the function f.

    (a) Determine the x values for which f has a horizontal tangent.

    (b) Determine the interval(s) where f is increasing.

    (c) Find the x values of all local maxima of f.

    (d) Determine the interval(s) where f is concave up.

    (e) Find the x values of any point(s) of inflection of f.

11. Let $f(x) = \dfrac{x^2 + x + 1}{(x+1)^2}$, so that

    $$f'(x) = \frac{x-1}{(x+1)^3} \quad \text{and} \quad f''(x) = \frac{2(2-x)}{(x+1)^4}.$$

    (a) Find the domain

    (b) Calculate the $y-$intercept of f.

    (c) Calculate the horizontal asymptote(s), if it exists.

    (d) Calculate the vertical asymptote(s), if it exists.

    (e) Determine where f is increasing and where f is decreasing. Label answers.

    (f) Find all local extrema of f.

    (g) Determine where f is concave up and where f is concave down.

    (h) Find all points of inflection.

    (i) Sketch the graph of f on the blank sheet provided, clearly indicating all of the information obtained above.

ANSWERS

1. (a) ∞
   (b) $\dfrac{7}{3}$
   (c) $\dfrac{\sqrt{2}}{2}$
2. $x = 0, \quad x = \dfrac{-3 \pm \sqrt{129}}{20}$
3. local min $= (0, 0)$, local max $= (8, 4)$
4. abs max $= 37$, abs min $= -27$
5. increasing: $\left(0, \dfrac{\pi}{6}\right) \cup \left(\dfrac{5\pi}{6}, 2\pi\right)$

   decreasing: $\left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$
6. concave down: $\left(0, \dfrac{2\pi}{3}\right) \cup \left(\dfrac{4\pi}{3}, 2\pi\right)$

   concave up: $\left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right)$
7. $x = 140$
8. cost $= \$163.54$
9. see instructor
10. see instructor
11. (a) $x \neq -1$
    (b) $(0, 1)$
    (c) $y = 1$
    (d) $x = -1$
    (e) increasing: $(-\infty, -1) \cup (1, \infty)$; decreasing: $(-1, 1)$
    (f) local min $= (1, 3/4)$
    (g) concave up: $(-\infty, -1) \cup (-1, 2)$; concave down: $(2, \infty)$
    (h) $\left(2, \frac{7}{9}\right)$
    (i) see instructor