- A function f is **continuous at a number** a if $\lim_{x \to a} f(x) = f(a)$. Note that in order for a function f to be continuous at a, three things must happen:

    1. $f(a)$ must be defined.
    2. $\lim_{x \to a} f(x)$ must exist.
    3. These two things must be the same; namely, $\lim_{x \to a} f(x) = f(a)$.

- A function f is **discontinuous at a number** a if f is not continuous at a.

- Intuitively, f is discontinuous at a if there is a hole, break, or jump in the graph at $x = a$.

- A function f is **continuous from the right at a number** a if $\lim_{x \to a^+} f(x) = f(a)$.

- A function f is **continuous from the left at a number** a if $\lim_{x \to a^-} f(x) = f(a)$.

Example 1: Determine if the following functions are continuous or discontinuous at the given point a. If it is discontinuous at a, state which condition fails.

- $f(x) = \begin{cases} x - 2 & \text{if } x \leq 3 \\ 5x + 1 & \text{if } x > 3 \end{cases}$ $\qquad a = 3$

- $g(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$ $\qquad a = 2$

- $h(x) = \begin{cases} 3x - 2 & \text{if } x < -1 \\ 4x^2 + 9x & \text{if } x \geq -1 \end{cases}$ $\qquad a = -1$

- A function f is **continuous on an interval** if it is continuous at every number in the interval. If f is defined only on one side of an endpoint, we understand that continuous at that endpoint to mean continuous from the right or continuous from the left.

- If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

$$f+g \qquad f-g \qquad cf \qquad fg \quad \text{and} \quad \frac{f}{g} \quad \text{if } g(a) \neq 0.$$

- If g is continuous at a and f is continuous at $g(a)$, then $f \circ g(x) = f(g(x))$ is continuous at a.

- Any polynomial is continuous everywhere.

- Any rational function is continuous wherever it is defined; that is, it is continuous on its domain.

- The following functions are continuous at every number in their domains: polynomials, rational functions, root functions, trigonometric functions.

- If f is continuous at b and $\lim_{x\to a} g(x) = b$, then $\lim_{x\to a} f(g(x)) = f(b)$. In other words,

$$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$$

Example 2: Determine the intervals where the following functions are continuous.

1. $f(x) = x^2 + x + 1$

2. $g(x) = \dfrac{2x+1}{x^2+5x+6}$

3. $h(x) = \sqrt{x} + \dfrac{x}{x-7}$

4. $f(x) = \tan 3x$

5. $g(x) = \dfrac{1}{\sqrt{x^2+5}-3}$

6. $h(x) = \sin x^4 + \cos(9x-5)$

Example 3: Locate all the discontinuities for

$$f(x) = \frac{9}{\sqrt{3} + 2\sin 3x}$$

Intermediate Value Theorem: Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

Example 4: Use the Intermediate Value Theorem to show that there is a root of $\tan x = 2x$ in the interval $(0, 1.4)$.

Homework: pp 54–55; #3, 5, 9, 13–31 odd, 37, 39 (You do not need to sketch any graphs.)