

- The **derivative of a function f at a number a** , denoted by $f'(a)$ is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

- **Other notations for the derivative**

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$

and

$$f'(a) = \left. \frac{dy}{dx} \right|_{x=a}$$

- A function f is **differentiable at a** if $f'(a)$ exists. It is **differentiable on an open interval (a, b)** (or (a, ∞) , $(-\infty, a)$, or $(-\infty, \infty)$) if it is differentiable at every number in the interval.
- **Theorem:** If f is differentiable at a , then f is continuous at a .
- The converse of the above theorem is NOT true.
- A function f is not differentiable at $x = a$ if
 1. the graph of the function f has a corner, kink, or sharp point at $x = a$.
 2. the function f is not continuous at $x = a$.
 3. the function f has a vertical tangent line at $x = a$