• The **derivative of a function** \( f \) **at a number** \( a \), denoted by \( f'(a) \) is

\[
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
\]

if this limit exists.

• **Other notations for the derivative**

\[
f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)
\]

and

\[
f'(a) = \frac{dy}{dx} \bigg|_{x=a}
\]

• A function \( f \) is **differentiable at** \( a \) if \( f'(a) \) exists. It is **differentiable on an open interval** \((a, b)\) (or \((a, \infty)\), \((-\infty, a)\), or \((-\infty, \infty))\) if it is differentiable at every number in the interval.

• **Theorem:** If \( f \) is differentiable at \( a \), then \( f \) is continuous at \( a \).

• The converse of the above theorem is NOT true.

• A function \( f \) is **not** differentiable at \( x = a \) if

1. the graph of the function \( f \) has a corner, kink, or sharp point at \( x = a \).
2. the function \( f \) is not continuous at \( x = a \).
3. the function \( f \) has a vertical tangent line at \( x = a \)