

Find all relative extrema of the following functions. You may use the first or second derivative test (when applicable).

1. $f(x) = -x^3 + 3x^2 - 2$

2. $f(x) = 6x - x^2$

3. $f(x) = (x - 5)^2$

4. $f(x) = x^3 - 3x^2 + 3$

5. $f(x) = x^4 - 4x^3 + 2$

6. $f(x) = x^{\frac{2}{3}} - 3$

7. $f(x) = x + \frac{4}{x}$

8. $f(x) = x^3 - 12x$

9. $f(x) = x^3 - 6x^2 + 12x - 8$

10. $f(x) = \frac{1}{4}x^4 - 2x^2$

11. $f(x) = (x - 1)(x + 2)^2$

12. $f(x) = x\sqrt{x + 3}$

13. $f(x) = \frac{4}{1 + x^2}$

14. $f(x) = \frac{24}{x^2 + 12}$

15. $f(x) = \frac{x^2 - 1}{2x + 1}$

16. $f(x) = \frac{x^2 + 1}{x^2 - 1}$

17. $f(x) = \sin x$

18. $f(x) = x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$

19. $f(x) = \cos x - x, \quad 0 \leq x < 2\pi$

20. $f(x) = \sin(x/2), \quad 0 \leq x < 4\pi$

21. $f(x) = \sec\left(x - \frac{\pi}{2}\right), \quad 0 < x < 4\pi$

22. $f(x) = 2 \sin x + \sin(2x), \quad 0 \leq x < 2\pi$

ANSWERS

1. Rel. min. at $x = 0$, $f(0) = -2$
Rel. max. at $x = 2$, $f(2) = 2$
2. Rel. max. at $x = 3$, $f(3) = 9$
3. Rel. min. at $x = 5$, $f(5) = 0$
4. Rel. max. at $x = 0$, $f(0) = 3$
Rel. min. at $x = 2$, $f(2) = -1$
5. Rel. min. at $x = 3$, $f(3) = -25$
6. Rel. min. at $x = 0$, $f(0) = -3$
7. Rel. max. at $x = -2$, $f(-2) = -4$
Rel. min. at $x = 2$, $f(2) = 4$
8. Rel. max. at $x = -2$, $f(-2) = 16$
Rel. min. at $x = 2$, $f(2) = -16$
9. No relative extrema
10. Rel. max. at $x = 0$, $f(0) = 0$
Rel. min. at $x = -2$, $f(-2) = -4$ and
 $x = 2$, $f(2) = -4$
11. Rel. max. at $x = -2$, $f(-2) = 0$
Rel. min. at $x = 0$, $f(0) = -4$
12. Rel. min. at $x = -2$, $f(-2) = -2$
13. Rel. max. at $x = 0$, $f(0) = 4$
14. Rel. max. at $x = 0$, $f(0) = 2$
15. No relative extrema
16. Rel. max. at $x = 0$, $f(0) = -1$
17. Rel. max at $x = \pi/2 + 2\pi n$, $f(\pi/2 + 2\pi n) = 1$
Rel. min at $x = 3\pi/2 + 2\pi n$, $f(3\pi/2 + 2\pi n) = -1$
18. Rel. min at $x = 1$, $f(1) = -2$
19. No Relative extrema
20. Rel. min at $x = 3\pi$, $f(3\pi) = -1$
Rel. max at $x = \pi$, $f(\pi) = 1$
21. Rel. min at $x = \pi/2$, $f(\pi/2) = 1$ and
 $x = 5\pi/2$, $f(5\pi/2) = 1$
Rel. max at $x = 3\pi/2$, $f(3\pi/2) = -1$ and
 $x = 7\pi/2$, $f(7\pi/2) = -1$
22. Rel. min at $x = 5\pi/3$, $f(5\pi/3)$
Rel. max at $x = \pi/3$, $f(\pi/3)$