MATH 12002 **Limit Laws** **Section 1.4**

Suppose that c is a constant and the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then

- $\lim_{x \to a} c = c$
- $\lim_{x \to a} x = a$
- $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
- $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
- $\lim_{x \to a} [f(x)g(x)] = \left[\lim_{x \to a} f(x)\right] \cdot \left[\lim_{x \to a} g(x)\right]$
- $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$
- $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer.
- $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer. If n is even, we assume that $\lim_{x \to a} f(x) > 0$.
- **Direct Substitution Property:** If f is a polynomial or a rational function and a is in the domain of f, then
$$\lim_{x \to a} f(x) = f(a).$$
- If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$, provided the limits exist.
- $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = L = \lim_{x \to a^-} f(x)$.

Examples: Evaluate the limit, if it exists.

1. $\lim_{x \to -4} \dfrac{x^2 + 5x + 4}{x^2 + 3x - 4}$

2. $\lim_{x \to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

3. $\lim_{h \to 0} \dfrac{(2 + h)^3 - 8}{h}$

4. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

5. $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$

Squeeze Theorem: If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

then $\lim_{x\to a} g(x) = L$

Example: Show that $\lim_{x\to 0} x^2 \sin \frac{\pi}{x} = 0$.

Homework: pp 43–44; #1, 2, 3, 11-23 odd, 29, 31, 37a