- Let f be a function defined on both sides of a, except possibly at a. Then

$$\lim_{x\to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large by taking x sufficiently close to a, but not equal to a.

- The line $x = a$ is called a **vertical asymptote** of $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x\to a} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty \qquad \lim_{x\to a^+} f(x) = \infty$$
$$\lim_{x\to a} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty \qquad \lim_{x\to a^+} f(x) = -\infty$$

Example 1: Find the limit.

1. $\displaystyle\lim_{x\to 5^+} \frac{6}{x-5}$

2. $\displaystyle\lim_{x\to \pi^-} \cot \pi$

- Let f be a function defined on some interval (a, ∞). Then $\lim_{x\to\infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

- Let f be a function defined on some interval $(-\infty, a)$. Then $\lim_{x\to-\infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large negative.

- The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = L \qquad \text{or} \qquad \lim_{x\to-\infty} f(x) = L$$

STEPS TO EVALUATE THE LIMIT AT INFINITY OF RATIONAL FUNCTIONS.

1. Divide both the numerator and denominator by the highest power of x that occurs in the denominator. (Note that since we are only interested in large values of x, we can assume that $x \neq 0$.)
2. Next, use the following property: If n is a positive number, then

$$\lim_{x \to \infty} \frac{1}{x^n} = 0 \qquad \text{and} \qquad \lim_{x \to -\infty} \frac{1}{x^n} = 0$$

3. Evaluate the limit.

EXAMPLES: Find the limit.

1. $\displaystyle \lim_{x \to \infty} \frac{3x^2 + 2}{7x^2 + x - 1}$

2. $\displaystyle \lim_{x \to -\infty} \frac{\sqrt{8x^4 - 1}}{x^2 + 1}$

3. $\lim_{x \to \infty} \left(\sqrt{x^4 + 6x^2} - x^2 \right)$

4. $\lim_{x \to -\infty} \left(x + \sqrt{x^2 + 2x} \right)$

5. $\lim_{x \to \infty} \left(\sqrt{x^2 + 1} - x \right)$

6. $\lim_{x\to-\infty} \cos x$

7. $\lim_{x\to-\infty} (x^2 - x^4)$

8. $\lim_{x\to\infty} \dfrac{x^3 - 2x + 3}{5 - 2x^2}$

Homework: pp 66–67; #1, 2, 3–7 odd, 13–31 odd