MATH 12002 **LIMITS** **Section 1.3**

- The function f has **limit** L as x approaches a, denoted

$$\lim_{x\to a} f(x) = L,$$

means that we can make $f(x)$ as close to L as we like by making x sufficiently close to a, but not equal to a.

- The function f has a **right-hand limit** L as x approaches a, denoted

$$\lim_{x\to a^+} f(x) = L,$$

means we can make $f(x)$ as close to L as we like by taking x sufficiently close, but not equal, to a and x to the right of a.

- The function f has a **left-hand limit** L as x approaches a, denoted

$$\lim_{x\to a^-} f(x) = L,$$

means we can make $f(x)$ as close to L as we like by taking x sufficiently close, but not equal, to a and x to the left of a.

- $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^+} f(x) = L = \lim_{x\to a^-} f(x)$

- Let f be a function defined on both sides of a, except possibly at a. Then

$$\lim_{x\to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large by taking x sufficiently close to a, but not equal to a.

- The line $x = a$ is called a **vertical asymptote** of $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x\to a} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty \qquad \lim_{x\to a^+} f(x) = \infty$$
$$\lim_{x\to a} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty \qquad \lim_{x\to a^+} f(x) = -\infty$$