**Newton's Method** is a generalized process to find an accurate root of a system (or a single) equations $f(x) = 0$.

Let $f$ be a differentiable function. Choose a point $x_1$ near a root of $f$. Define recursively

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

If the point $x_1$ is chosen sufficiently close to the root then the $x_n$'s are successively better approximations of the root.

EXAMPLE: Find the root of $x^4 + x - 4 = 0$ in the interval $[1, 2]$ accurate to 6 decimal places.

SOLUTION: Note that the root will be accurate to 6 decimal places when $x_n$ and $x_{n+1}$ agree to 6 decimal places.

For this problem $f(x) = x^4 + x - 4$ and therefore, $f'(x) = 4x^3 + 1$. Choose $x_1 = 1.5$. The iterative process is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 + x_n - 4}{4x_n^3 + 1}.$$

Beginning the iterative process, we have $x_1 = 1.5$ and

$$\begin{aligned} x_2 &= 1.5 - \frac{1.5^4 + 1.5 - 4}{4(1.5)^3 + 1} \\ &= 1.323275862. \end{aligned}$$

Next, we have

$$\begin{aligned} x_3 &= 1.323275862 - \frac{(1.323275862)^4 + 1.323275862 - 4}{4(1.323275862)^3 + 1} \\ &= 1.285346065. \end{aligned}$$

$$\begin{aligned} x_4 &= 1.285346065 - \frac{(1.285346065)^4 + 1.285346065 - 4}{4(1.285346065)^3 + 1} \\ &= 1.283784219. \end{aligned}$$

$$\begin{aligned} x_5 &= 1.283784219 - \frac{(1.283784219)^4 + 1.283784219 - 4}{4(1.283784219)^3 + 1} \\ &= 1.283781666. \end{aligned}$$

$$\begin{aligned} x_6 &= 1.283781666 - \frac{(1.283781666)^4 + 1.283781666 - 4}{4(1.283781666)^3 + 1} \\ &= 1.283781666. \end{aligned}$$

Now, $x_5$ and $x_6$ agree to 6 decimal places, so the root accurate to 6 decimal places is 1.283781.