SECTION 2.7—RELATED RATES

GUIDELINES FOR SOLVING RELATED RATES PROBLEMS

1. Read the problem carefully.
2. Draw a diagram if possible.
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express the given information and the required rate in terms of derivatives.
5. Write an equation that relates the various quantities of the problem. If necessary, use the geometry of the situation to eliminate one of the variables by substitution.
6. Differentiate both sides of the equation with respect to t (probably using implicit differentiation.)
7. Substitute the given information into the resulting equation and solve for the unknown rate.

SOLVE THE FOLLOWING RELATED RATES PROBLEMS:

1. Suppose the radius of a circle is increasing at $7\,\text{cm/sec}$. How fast is the area increasing when the radius is $20\,\text{cm}$?

2. The sides of a square are increasing at the rate of $2\,\text{in/min}$. At what rate is the area of the square increasing when the sides are $4\,\text{in}$?

3. Gas is pumped into a spherical balloon at the rate of $1\,\text{ft}^3/\text{min}$. How fast is the radius of the balloon increasing when the balloon contains $36\,\text{ft}^3$ of gas?

4. A painter is painting a house using a ladder 15 feet long. A dog runs by the ladder dragging a leash that catches the bottom of the ladder and drags it directly away from the house at 22 ft/sec. Assuming that the ladder continues to be pulled away at this speed, how fast is the top of the ladder moving down the wall when the top is 5 feet from the ground?

5. An angler has a fish at the end of a line. The line is reeled in at 2 ft/sec from a bridge 30 feet above the waterline. At what rate is the fish moving through the water when the length of the line is 50 feet?

6. A camera televising the return of the opening kickoff of a football game is located 5 yd from the east edge of the field and in line with the goal line. The player with the football runs down the east edge (just in bounds) for a touchdown. When he is 10 yd from the goal line, the camera is turning at a rate of 0.5 radian/sec. How fast is the player running?

7. Water is poured into a conical paper cup at the rate of 2/3 cubic inches per second. If the cup is 6 inches tall and the top of the cup has a radius of 2 inches, how fast does the water level rise when the water is 4 inches deep?