Definition: If $f$ is continuous function defined for $a \leq x \leq b$, we divide $[a, b]$ into $n$ subintervals of equal width $\Delta x = \dfrac{b-a}{n}$. We let $x_0 = a,\ x_1,\ x_2,\ \ldots,\ x_n = b$ be the endpoints of these subintervals. Next, we choose sample points $x_1^*,\ x_2^*,\ \ldots,\ x_n^*$ in these subintervals, so $x_i^*$ lies in the $i$-th subinterval $[x_{i-1}, x_i]$. Then the **definite integral of** $f$ from $a$ to $b$ is

$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

Remarks:

1. $\int$ is called the integral, and $f(x)$ is called the integrand. $a$ is called the lower limit of integration and $b$ is the upper limit of integration. $\sum_{i=1}^{n} f(x_i^*)\Delta x$ is called the **Riemann Sum**.
2. $\int_a^b f(x)\ dx$ is a number.
3. Because $f$ is continuous, it can be proven that the limit in the definition exists and always gives the same value no matter how we choose the sample points $x_i^*$.
4. If $f$ is positive, then $\int_a^b f(x)\ dx$ is the area under the curve.
5. If $f$ has both positive and negative values, then

$$\int_a^b f(x)\ dx = (\text{area above x-axis}) - (\text{area below x-axis}).$$