MATH 12003 — TRIGONOMETRIC FORMULAS

EVALUATION OF $\int \sin^m x \cos^n x \, dx, \quad m \geq 0, n \geq 0$

Condition	Useful Identity	Method
1) n odd	$\cos^2 x = 1 - \sin^2 x$	Substitute $u = \sin x$
2) m odd	$\sin^2 x = 1 - \cos^2 x$	Substitute $u = \cos x$
3) m and n odd		Choose 1 if $m > n$ Choose 2 if $m < n$ Choose either if $m = n$
4) m and n even	Use Identities 4, 5, or 6 below	Reduce to smaller powers

EVALUATION OF $\int \tan^m x \sec^n x \, dx, \quad m \geq 0, n > 0$

Condition	Useful Identity	Method
1) n even	$\sec^2 x = 1 + \tan^2 x$	Substitute $u = \tan x$
2) m and n odd	$\tan^2 x = \sec^2 x - 1$	Substitute $u = \sec x$
3) m even, n odd		Reduce to powers of $\sec x$

TRIGONOMETRIC IDENTITIES

1. $\sin^2 x + \cos^2 x = 1$
2. $1 + \tan^2 x = \sec^2 x$
3. $1 + \cot^2 x = \csc^2 x$
4. $\sin(2x) = 2 \sin x \cos x$
5. $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
6. $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$
7. $\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$
8. $\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$
9. $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

TRIGONOMETRIC INTEGRALS

1. $\int \tan u \, du = \ln|\sec u| + C$
2. $\int \cot u \, du = \ln|\sin u| + C$
3. $\int \sec u \, du = \ln|\sec u + \tan u| + C$
4. $\int \csc u \, du = -\ln|\csc u + \cot u| + C$
5. $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin(2u) + C$
6. $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin(2u) + C$
7. $\int \tan^2 u \, du = \tan u - u + C$
8. $\int \cot^2 u \, du = -\cot u - u + C$
9. $\int \sec^3 u \, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$
10. $\int \sin^4 u \, du = -\frac{1}{4}\sin^3 u \cos u - \frac{3}{8}\sin u \cos u + \frac{3}{8}u + C$
11. $\int \cos^4 u \, du = \frac{1}{4}\cos^3 u \sin u + \frac{3}{8}\cos u \sin u + \frac{3}{8}u + C$
12. $\int \frac{1}{u^2 + a^2} \, du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$