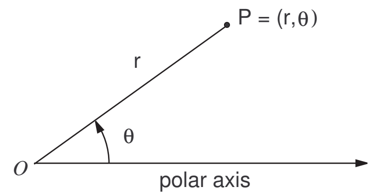
MATH 12003 CALCULUS AND POLAR COORDINATES Section 11.3

Below is the **polar coordinate system**:



- If P is any point in the plane, let r be the distance from O to P and let θ be the angle between the polar axis and the line OP. Then the point P is represented by the ordered pair (r, θ) and $r,\ \theta$ are called the **polar coordinates** of P.

- θ is positive if measured in the counter clockwise direction and negative if measured in the clockwise direction.

- The points $(-r, \theta)$ and (r, θ) lie on the same line through O and at the same distance $|r|$ from O, but on the opposite sides of O. Thus, $(-r, \theta)$ represents the same point as $(r, \theta+\pi)$.

- The points in the plane do not have a unique polar representation. The point (r, θ) can also be represented by

$$(r, \theta + 2n\pi) \qquad \text{or} \qquad (-r, \theta + (2n+1)\pi)$$

- If the point P has Cartesian coordinates (x, y) and polar coordinates (r, θ), then we have

$$x = r\cos\theta \qquad \text{and} \qquad y = r\sin\theta.$$

  This allows us to find the Cartesian coordinates of a point when the polar coordinates are known.

- To find r and θ when x and y are known, we use the equations

$$r^2 = x^2 + y^2 \qquad \text{and} \qquad \tan\theta = \frac{y}{x}.$$

  CAUTION: choose θ so that the point (r, θ) lies in the correct quadrant.

EXAMPLE: Plot the following points.

(a) $\left(1, \frac{\pi}{3}\right)$

(b) $\left(3, \frac{3\pi}{4}\right)$

(c) $\left(2, -\frac{\pi}{6}\right)$

(d) $\left(-2, \frac{\pi}{4}\right)$

(e) $\left(-1, -\frac{2\pi}{3}\right)$

EXAMPLE 2: Find the Cartesian coordinates of the following:

(a) $\left(2, \frac{2\pi}{3}\right)$

(b) $\left(-3, \frac{\pi}{6}\right)$

EXAMPLE 3: Find the polar coordinates of the following:

(a) $(1, 1)$

(b) $\left(-\sqrt{3}, 1\right)$

Homework: pp 713–714; #1–5 odd, #15–25 odd

Graphing Polar Equations

The **graph of a polar equation** $r = f(\theta)$ consists of all points P that have at least one polar representation (r, θ) whose coordinates satisfy the equation.

EXAMPLE 1: Graph the following:

(a) $r = 3$

(b) $r = 2\sin\theta$

(c) $\theta = \dfrac{\pi}{r}$

(d) $r = 2 + 2\sin\theta$

(e) $r = \cos 2\theta$

(f) $r = 1 + 2\cos\theta$

Homework: pg 714; #29–45 every other odd (eoo)

Tangents to Polar Curves

To find a tangent line to a polar curve $r = f(\theta)$ we regard θ as a parameter and write its parametric equations as

$$x = r\cos\theta = f(\theta)\cos\theta \qquad \text{and} \qquad y = r\sin\theta = f(\theta)\sin\theta.$$

Using the above equations, we have

$$\frac{dy}{dx} = \frac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \frac{\dfrac{dr}{d\theta}\sin\theta + r\cos\theta}{\dfrac{dr}{d\theta}\cos\theta - r\sin\theta}$$

Therefore, we can make the following observations:

- Solutions to $\dfrac{dy}{d\theta} = 0$ yield horizontal tangents, provided that $\dfrac{dx}{d\theta} \neq 0$.
- Solutions to $\dfrac{dx}{d\theta} = 0$ yield vertical tangents, provided that $\dfrac{dy}{d\theta} \neq 0$.
- When both $\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ are zero, we need to use a limit to determine what happens at that value of θ.

EXAMPLE 1: Find the points (in polar form) on the curve $r = 2(1 - \cos\theta)$ that have vertical or horizontal tangents.

Homework: pg 714; #55–65 odd