PARABOLAS

A **parabola** is the set of points in a plane that are equidistant from a fixed point F, called the **focus**, and a fixed line, called the **directrix**.

Equation of a parabola:

An equation of the parabola with focus $(0, p)$ and directrix $y = -p$ is

$$y = \frac{1}{4p}\, x^2 \tag{1}$$

NOTE: The parabola opens up if $p > 0$ and down if $p < 0$.

An equation of the parabola with focus $(p, 0)$ and directrix $x = -p$ is

$$x = \frac{1}{4p}\, y^2 \tag{2}$$

NOTE: The parabola opens to the right if $p > 0$ and to the left if $p < 0$.

- A parabola with equation of either form (1) or (2) is said to be in **standard position**.
- The point halfway between the focus and the directrix is the **vertex** of the parabola.
- The line through the focus that is perpendicular to the directrix is the **axis of symmetry** of the parabola.

ELLIPSES

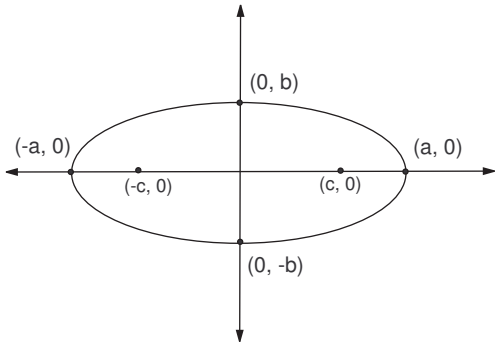
An **ellipse** is the set of points in the plane the sum of whose distances from two fixed points F_1 and F_2 is a constant. F_1 and F_2 are called the foci.

Equation of an ellipse:

The ellipse of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a \geq b > 0 \tag{3}$$

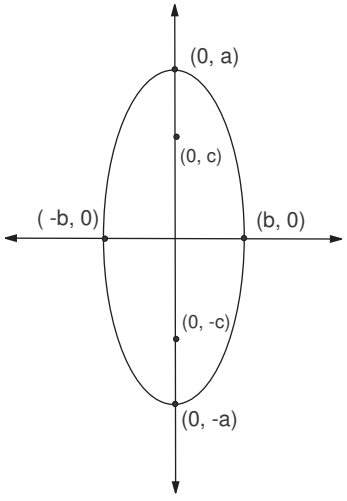
has foci $(\pm c, 0)$, where $c^2 = a^2 - b^2$, and vertices $(\pm a, 0)$.



The ellipse of the form

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \qquad a \geq b > 0 \tag{4}$$

has foci $(0, \pm c)$, where $c^2 = a^2 - b^2$, and vertices $(0, \pm a)$.



NOTE: The line segment connecting the vertices is called the **major axis**.

HYPERBOLAS

A **hyperbola** is the set of all points in a plane the difference of whose distances from two fixed points F_1 and F_2 is a constant. F_1 and F_2 are called the foci.

Equation of a hyperbola:

The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{5}$$

has foci at $(\pm c, 0)$, where $c^2 = a^2 + b^2$, vertices $(\pm a, 0)$, and asymptotes $y = \pm \frac{b}{a}\ x$.

The hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \tag{6}$$

has foci at $(0, \pm c)$, where $c^2 = a^2 + b^2$, vertices $(0, \pm a)$, and asymptotes $y = \pm \frac{a}{b}\ x$.

EXAMPLE 1: Find vertex (or vertices), focus (or foci), and asymptotes (if applicable) for the following conic sections.

1. $9x^2 - 4y^2 = 36$

2. $4x^2 + 25y^2 = 25$

3. $y^2 = 12x$

EXAMPLE 2: Find an equation for the conic that satisfies the given conditions.

1. Parabola with focus $(3, 6)$, vertex $(3, 2)$

2. Ellipse with foci $(0, -1)$, $(8, -1)$, vertex $(9, -1)$

3. Hyperbola with foci $(2, -2)$, $(2, 8)$, vertices $(2, 0)$, $(2, 6)$