Theorem 1. If f has a power series representation at a, that is, if

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \qquad |x-a| < R$$

then its coefficients are given by the formula $c_n = \dfrac{f^{(n)}(a)}{n!}$

Definition: The **Taylor Series of f about** a is given by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Definition: When $a = 0$, this series is called the **Maclaurin series of** f:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

(Hence, the Maclaurin series is just the Taylor series of f about $x = 0$.)

Important Maclaurin Series (page 803)

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \qquad \text{for } |x| < 1.$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n, \qquad \text{for all } x.$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1}, \qquad \text{for all } x.$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{2n}, \qquad \text{for all } x.$$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{2n+1}, \qquad \text{for } |x| \le 1.$$

$$\ln(x+1) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} x^{n+1}, \qquad \text{for } |x| < 1.$$