**The Integral Test:** Suppose $f$ is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ is convergent. That is,

- If $\int_1^{\infty} f(x)\,dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.
- If $\int_1^{\infty} f(x)\,dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

**Important notes about the integral test**

- It is not necessary for the integral or the sum to start at 1.
- It is not necessary that the function $f$ be always decreasing, just that it is decreasing for all $x$ larger than some number $N$.
- The sum of the series does not equal the value of the integral.

$p$**-series**: The $p$-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and is divergent if $p \leq 1$.

EXAMPLES:

1. $\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges since it is a $p$-series with $p = 5$.

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges since it is a $p$-series with $p = 1/2$.

EXAMPLES: Determine whether the series is convergent or divergent.

1. $\displaystyle\sum_{n=5}^{\infty} \frac{7}{n-4}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n+1}$

3. $\displaystyle\sum_{n=6}^{\infty} \frac{1}{n^2-4n+5}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{3n+2}{n(n+1)}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

6. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n \ln n \ln(\ln n)}$